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Evolution of system noises in electrodeposition process control

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ABSTRACT

The evolution of source noises in space–time is analysed in this paper. It characterises the global effect of uncertainties in electrodeposition process control, where the source noises have an effect on the concentration field of relevant species in the diffusion layer and the field is controlled by the Neumann boundary using relatively simple boundary controls. The control errors evolve in the diffusion layer and are dependent upon the source noises and applied controls as a random field process. The covariance structure of the field is found analytically and confirmed numerically. The local source noises are incited by the uncertainties from a realistic control system; they are devised by the process physics and a control system structure. This paper demonstrates that even in a relatively simple system, the local uncertainties have a strong tendency to expand in space–time. Some source noises have a dispersed effect on the overall system uncertainty (control error), others are more local and do not expand in the same way. The noise of the mass flux, which is injected through the Neumann boundary, dies out quickly in the diffusion layer.

1. Introduction

Due to a lack of complete knowledge about mass transfer and surface reactions as well as incomplete observations pertaining to electrodeposition process control, a stochastic model should be applied. Such a model has been developed by Tenno [1] based on his analysis of the uncertainties in electrodeposition process control. This process has five categories of uncertainties modelled as source-noises in the single dimensional domain (diffusion layer) and on the boundaries (one end: cathode, other: bulk solution). While existence of the local source noises can be understood based on process physics and implementation, their global effect on the concentration field is mostly unknown. The spatial effect of local noises is widespread. The overall effect of all categories of noises on the concentration field of relevant species in the diffusion layer is a rather complex stochastic evolution process in space–time. The purpose of this paper is to isolate a particular effect of each source noise on the concentration field that characterises the overall effect of uncertainties on the control system. The process is controlled through one boundary by adjusting the electric current flow through the cathode. A control error arising during the electrodeposition process expands in space–time and is dependent upon the local source noises and boundary controls that are applied.

In this paper, the covariance structure of system noises will be derived using several steps and then verified numerically. The idea of numerical analysis is simple. The stochastic system will be compared with the corresponding noise-free system; the difference between them will reveal the effect of noise on the concentration field in space–time. The main statistics can then be evaluated spatially and temporarily by the field differences, which are dependent upon the applied source noises (case by case) and boundary controls. Deriving the covariance structure is more of a laborious process. At first, the noise-free system with relatively simple boundary controls will be solved analytically. These feedback controls can be represented in the feed-forward controls using an analytical solution for the system on the boundary. The feed-forward controls turn out to be a smooth enough function for the next step, which involves homogenising the stochastic system on the boundary. In addition to the feed-forward control, a stochastic modification of the noises is used to homogenise the system, which will eventually represent a Gaussian process. The solution for the homogenised system is expressed through the variation of constants formula as a mild solution using a Green's function in an explicit form. Ultimately, the spatial covariance and temporal covariance will be derived on the basis of the mild solution. The mean value of the concentration field coincides with the noise-free (deterministic) field.

In general theory, a covariance trace is analysed instead of the covariance function for a field. This theory deals with Ito's differentiation rule and the Kolmogorov equation in infinite dimensions. The function for a change of variable using Ito's formula and the initial function in the Kolmogorov equation are assumed to be an integral-type bounded function that maps a Hilbert space onto a one-dimensional Euclidian space. In this application-limiting condition, a covariance trace can be derived from Ito's formula in infinite dimensions [2] as an integro-differential equation that can

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be further simplified with respect to the noise-related terms since a finite number of Wiener processes are applied in our model. Although the covariance trace can be found as a formal solution to a certain operator equation, its properties are still unknown. In a more general sense, they have been investigated as part of a general theory dealing with the infinite dimensional processes induced by a cylindrical Wiener process [3–5]. In the latter case, the covariance trace can be found based on a probability density that satisfies a Fokker–Plank equation, or, in a more general sense, satisfies a Kolmogorov equation in infinite dimensions (e.g. [5–7] and the references therein). Both these equations have been comprehensively analysed, especially in finite dimensions [8], and, somewhat less often, in infinite dimensions, and the existence and uniqueness of the variational solution as well as the mild solution have been verified. There is a vast literature on the processes considered in the entire space (e.g. [5,7] and the references therein). Similar results for a bounded domain with homogeneous Dirichlet boundary conditions have been proposed in other studies (e.g. [9]), but such results are lacking in more practical cases for an inhomogeneous mixed noise boundary and for the controls exercised through the boundary, like those considered in this paper. Although our finite-dimensional Wiener process induces more regular behaviour during process evolution than a cylindrical Wiener process, the benefit of simpler noises is not stressed in the literature. In this light, the result presented by Krylov [10] is encouraging. He was able to show that a Kolmogorov equation with finitely many state variables can be solved in the less limiting condition of polynomial growth that is applied to the initial function. However, the theory in general is not so encouraging. Techniques for solving the stochastic evolutionary equations with functional derivatives were for the most part not present in the existing literature. In contrast to the general theory, a more pragmatic approach is taken in this paper, one that deals with the covariance structure of a random field instead of the covariance trace considered in the existing literature.

2. Stochastic system for electrodeposition process control

A stochastic model that reflects the electrodeposition process uncertainties has been proposed by Tenno [1]. This model explains deposition as a diffusion process with stochastic noises in a one-dimensional diffusion layer when the process flux is measured and controlled on one end of the boundary and the bulk solution concentration level is fixed on the other end. All the processes in the domain interior and on the boundaries are corrupted by local source noises. Since this model has been described elsewhere [1], here it is summarised briefly in Sections 2.1 and 2.2 and then applied to the noise effect (control error) analysis throughout the rest part of the paper.

2.1. The system model

Assume that a consumption reaction of the relevant species takes place on a solid surface ($a = 0$) and that its mass transfer in a finite interval (domain), $\Omega = (a \le x \le b)$, is dominated by diffusion. In a real case scenario, a number of different types of uncertainties affect the process, and hence, a stochastic model is applied. In this model, the concentration of species is given by the stochastic diffusion equation:

$$dc = Ac(t,x)dt + \sigma(x)dW(t). \quad (1)$$

In Eq. (1), $c(t,x)$ is the concentration (mol/m^3) of the relevant species, A is the linear operator, $Ac = D_0 c_{xx} + k_0 c_x + k_r c$, which generates a strongly continuous semi-group that satisfies the coercivity and other standard assumptions, $D_0 > 0$, $k_0 \le 0$, $k_k \le 0$, that appear in the natural condition for the electrodeposition process, D_0 is the diffusivity (m^2/s) of species and $\sigma(x)dW(t)$ is a process noise. The initial condition for this system is that the concentration is initially on a bulk level: $c(0, x) = c_{bulk}$. Once $t > 0$, the mixed boundary data are given as a Neumann condition,

$$D_0 c_x(t,a) = u(t) + \sigma_0^a \dot{W}_a(t), \quad (2)$$

on boundary a (cathode) and as a Dirichlet condition,

$$c(t,b) = c_{bulk} + \sigma_0^b \dot{W}_b(t), \quad (3)$$

on boundary b (bulk end). Both these boundaries are corrupted with generalised white noises, $\dot{W}_a(t)$, $\dot{W}_b(t)$, with the given intensities σ_0^a, σ_0^b. Furthermore, in Eq. (2) $u(t)$ denotes our control variable, which is also the mass flux (mol/m^2/s) related to the reaction of the consumed species; in other units it represents the growth rate (m/s) of deposit. This is important to notice (in a stochastic context) because we can control and measure the rate that exists as a physical parameter. In practical systems, $u(t) = F_z^{-1} i(t)$ is manipulated by adjusting a current density, $i(t)$. F_z denotes a given coefficient, $F_z^{-1} = zF$, where z is the electron number of the consumed species and F is Faraday's constant, 96487C/mol.

From a system point of view, the concentration of species is known, $c(t, b) = c_{bulk}$, only on the outer boundary, b, of the diffusion layer – elsewhere it is not observed, including on the controlled boundary, $c(t,a)$, i.e. at the electrode surface. As mentioned above, the current related to the consumption reaction can be measured, but this only yields the mass flux over the boundary, and therefore, it does not indicate the concentration level there.

In practice, we measured a current density that had been corrupted by unmodelled side reactions, modelled here as a white noise, $\sigma_0^a \dot{W}_a(t)$, as well as a sensor noise:

$$D_0 \dot{\xi}(t) = u(t) + \sigma_0^a \dot{W}_a(t) + r\dot{V}(t). \quad (4)$$

The sensor noise, $r\dot{V}(t)$, is a generalised white noise with intensity r. The above-mentioned white noise terms on the boundaries, $\sigma_0^a \dot{W}_a(t), \sigma_0^b \dot{W}_b(t), r\dot{V}(t)$, which can be written in a differential form, $\sigma_0^a dW_a(t)$, and integrated over time, gain their exact meaning as Ito's integrals.

The domain noise. Due to the structure of the deposition process uncertainties, the domain noise, $\sigma(x)dW(t)$, can be decomposed in the boundary-related $\sigma^a(x), \sigma^b(x)$ components and domain-related $\sigma^d(x)$ component as follows:

$$\sigma(x) = \sqrt{(\sigma^a(x))^2 + (\sigma^b(x))^2 + (\sigma^d(x))^2}. \quad (5)$$

The domain noise is finite dimensional and is represented as a weighted sum of one-dimensional Wiener processes:

$$\sigma(x)dW(t) = \sum_{i=1}^{N} \chi_i \sigma(x_i) dW_i(t). \quad (6)$$

Here, the noise, $W_i(t)$, is applied point-wise in each point (x_i, $i = 1, \ldots, N$) of the partition for the interval (a, b): $W_i(t)$ is standard Wiener process and $\sigma(x_i)$ is the noise intensity or a weight function that characterises the standard deviation of constant noise in the interval $(x_{i-1}, x_i]$: this function is approximated as the step function, $\sigma_i = \sigma(x_i)\chi_i$, which uses the weight coefficients, $\sigma(x_i)$, and indicator functions, $\chi_i = 1_{(x_{i-1}, x_i]}(x)$. Similar to Eq. (5), the weights, σ_i, consist of three components, each of which is proportional to the coefficient ρ_k:

$$\sigma_i^a = \rho_a G(a, s_a)(x_i), \quad \sigma_i^b = \rho_b G(b, s_b)(x_i), \quad \sigma_i^d = \rho_d G(b, s_d)(x_i) \quad (7)$$

and Gaussian kernel, $G(k, s)(x) = (2/s\sqrt{2\pi}) e^{-((x-k)^2/2s^2)}$. In Eq. (7), the coefficient ρ_k scales the overall weight of the noises based on their types, $k = a$, b, d, and s_k causes the weight function, σ^k, to become concentrated at either boundary, $k = a$ or b. According to