Journal of Process Control 24 (2014) 146–151

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Control configuration selection for linear stochastic systems

Hamid Reza Shaker[a], Fatemeh Shaker[b,*]

[a] Faculty of Engineering and Natural Sciences, Aalesund University College, Ålesund, Norway
[b] Islamic Azad University of Kazeroon, Kazeroon, Iran

ARTICLE INFO

Article history:
Received 10 February 2013
Received in revised form 7 November 2013
Accepted 7 November 2013
Available online 15 December 2013

Keywords:
Control configuration selection
Stochastic systems
Process control
Controllability gramians
Observability gramian
Interaction measure

ABSTRACT

An appropriate control configuration selection is identified as one of the key prerequisites for attaining the control objectives in industrial practices. To select a suitable control configuration, it is important to determine which variables should be measured and how the process should be actuated. Therefore, the first step is to determine the optimal locations for the sensors and actuators. For the multivariable processes, this step is followed by choosing the appropriate input and output pairs for the design of SISO (or block) controllers. This is due to the popularity of the distributed and decentralized control in industrial control systems. These issues, which have been studied extensively for deterministic systems, have not been closely studied for stochastic systems. In this paper however the problem of control configuration selection is studied for the linear stochastic systems. The problem of selecting the sensor locations for stochastic systems is viewed as the problem of maximizing the output energy generated by a given state and for the actuator locations is viewed as the problem of minimizing the input energy required to reach a given state. Furthermore, a gramian-based interaction measure for control structure selection of multivariable stochastic systems is proposed. This interaction measure can be used to propose a richer (sparse or block diagonal) controller structure for distributed and partially decentralized control.

1. Introduction

With the ever increasing complexity of the process plants and manufacturing processes, the objectives of process control strategies cannot be attained unless a suitable control configuration is selected. To select an appropriate control configuration, it is important to determine which variables should be measured and how the process should be actuated. Therefore, the first step is to determine the optimal locations for the sensors and actuators. This makes providing the accurate and reliable process measurements and suitable actuations possible for the control purposes. For the multivariable processes, this step is followed by choosing the appropriate input and output pairs for the design of SISO (or block) controllers. This is due to the popularity of the distributed and decentralized control in industrial control systems. The reason for this popularity is that the centralized control of large-scale complex systems are expensive and difficult, due to the computational complexity, the problems related to reliability and the limitations in communications. On the other hand, decentralized controllers are easy to understand for operators, easy to implement and to re-tune [1,2].

In this paper both key issues in control configuration selection are addressed. These two key issues have been studied extensively for deterministic systems. For the placement of the sensors and the actuators, several techniques have been proposed over the last few decades. These techniques take into account different performance criteria [3–9]. One of the most reliable criterion for determining sensor and actuator locations is the improvement of state controllability and observability of the process [3]. In these methods, the problem of determining the sensor locations is viewed as the problem of maximizing the output energy generated by a given state. The problem for the actuator locations is viewed as the problem of minimizing the input energy required to reach a given state. In [4–6], several gramian-based methods from this category for optimal placement of the sensors and the actuators have been proposed. These methods have been improved and have been extended to unstable systems in [9] and further to nonlinear systems in [7,8].

The second key issue of control configuration selection which is input–output pairing or the controller structure selection, has also been studied extensively for multivariable deterministic systems. The results in this context are based on different interaction measures. Interaction measures make it possible to study input–output interactions and to partition a process into subsystems in order to reduce the coupling, to facilitate the control and to achieve a satisfactory performance. There are two broad categories of interaction measures in the literature. The first category is the relative gain array (RGA) and its related indices [10–16] and the second category is the family of the gramian-based interaction measures [17–26].

The relative gain array (RGA) is the most well-known interaction measure. It was first proposed in [10]. RGA uses d.c. gain of the process to quantify the channel interactions. The RGA can also

* Corresponding author.
E-mail address: f.shaaker@gmail.com (F. Shaker).

0959-1524/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jprocont.2013.11.008

be computed for a particular frequency other than zero. However, RGA is not sensitive to delays. The RGA has been studied by several other researchers (see, e.g. [11,12]). There are also other similar measures of interaction, which use dc gain of the process e. g. the NI (the Niederlinski index) [13]. The second category of the interaction measures is the family of the gramian based methods. A method from this category was first proposed in [17] and further in [18]. In this category, the observability and the controllability gramians are used to form the Participation Matrix (PM). The elements of the PM encode the information on the channel interactions. PM is used for pairing and the controller structure selection. The Hankel Interaction Index Array (HIIA) is a similar interaction measure, which was proposed in [19]. The gramian-based interaction measures have several advantages over the interaction measures in the RGA category. The gramian-based interaction measures take the whole frequency range or a bounded interval of frequency into account rather than a single frequency. This family of the interaction measures suggests more suitable pairing and allows more complicated controller structures. For more details on the applications and the differences between two main categories of the interaction measures, see [20,21].

It is apparent from the presented literature survey that these two important issues of control configuration selection, which have been studied extensively for deterministic systems, have not been closely studied for stochastic systems. In this paper however these key issues of control configuration selection are studied for the linear stochastic systems. The problem of selecting the sensor locations for stochastic systems is viewed as the problem of maximizing the output energy generated by a given state and for the actuator locations is viewed as the problem of minimizing the input energy required to reach a given state. Furthermore, a gramian-based interaction measure for control structure selection of multivariable stochastic systems is proposed. This interaction measure can be used to propose a richer sparse or block diagonal controller structure for distributed and partially decentralized control.

The paper is organized as follows. In Section 2, we introduce gramians for stochastic systems. The generalized Lyapunov equations and their solvability conditions and their energy interpretations are also discussed in this section. Section 3 presents the main results of the paper. This sections addresses both key issues of control configuration selection for stochastic systems. The methods are further illustrated with the help of a numerical example in Section 4 and the results are discussed. Finally, the last section concludes the paper.

The notation used in this paper is as follows: M^* denotes transpose of matrix if $M \in \mathbb{R}^{n\times m}$ and complex conjugate transpose if $M \in \mathbb{C}^{n\times m}$. The Moore–Penrose-inverse of M is denoted by $M^{\#}$. The $\otimes$ stands for the Kronecker Product. $Struc(\Pi) = \left[\pi_{ij}\right]_{p\times p}$ shows the structure of Π which is a symbolic array where π =*, if there exist a subsystem in Π with input u_j and output y_i. Otherwise: $\pi = 0$. The standard notation >, $\geq$(<, $\leq$) is used to denote the positive (negative) definite and semidefinite ordering of matrices.

2. Gramians and energy functionals for linear stochastic systems

The gramians are matrices with the embedded controllability and observability information. The controllability and observability gramians were first introduced in [27,28] for linear deterministic systems and more recently in [29]. It is well-known that the controllability gramian shows the level of controllability. Similarly, the observability gramian contains information of the level of observability for a system. The gramians have been largely used in the process of model reduction [29–32].

Let Σ be a stochastic linear control system of Itô-type [32,33], which is described by:

$$\Sigma : \begin{cases} dx = Ax\,dt + \sum_{j=1}^{N} A_j x d\omega_j + Bu\,dt \\ y = Cx \end{cases} \quad (1)$$

The $\omega_j = \omega_j(t)$ are independent zero mean real Wiener processes on the probability space $(\Omega, \mathcal{F}, \mu)$ with respect to an increasing family $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$.

Let $L^2_\omega(\mathbb{R}_+, \mathbb{R}^q)$ denote the corresponding space of non-anticipating stochastic processes υ with value in $\mathbb{R}^q$ and norm:

$$\left\|\upsilon(.)\right\|^2_{L^2_\omega} := \mathcal{E}\left(\int_0^\infty \left\|\upsilon(t)\right\|^2 dt\right) < \infty \quad (2)$$

where $\mathcal{E}$ denotes expectation. We assume that the homogeneous equation:

$$dx = Ax\,dt + \sum_{j=1}^{N} A_j x d\omega_j \quad (3)$$

is mean-square stable for all initial conditions $x(0) = x_0$. Let Φ be the fundamental solution for this equation, such that: $x(t) = \Phi(t, 0)x_0$. In general, stochastic differential equations can only be solved forward in time $\Phi(t, \tau)$ is only defined for $t \geq \tau$. From the time-invariance property, we have: $\Phi(t, \tau) = \Phi(t-\tau, 0)$. For simplicity we write: $\Phi(t) = \Phi(t, 0)$, where $t \geq 0$.

For the stochastic system (1), the controllability gramian P and the observability gramian Q are defined as [32]:

$$P := \mathcal{E}\left(\int_0^\infty \Phi(t)BB^*\Phi(t)^* dt\right) \quad (4)$$

$$Q := \mathcal{E}\left(\int_0^\infty \Phi(t)^*C^*C\Phi(t) dt\right) \quad (5)$$

These gramians are the solutions of the following generalized Lyapunov equations:

$$AP + PA^* + \sum_{j=1}^{N} A_j P A_j^* + BB^* = 0 \quad (6)$$

$$A^*Q + QA + \sum_{j=1}^{N} A_j^* Q A_j + C^*C = 0 \quad (7)$$

For a given $x_0 \in \mathbb{R}^n$, the minimal energy of an input u can be determined such that $\mathcal{E}(x(T, 0, u)) = x_0$ for some $T > 0$. Dually, the output energy produced by x_0 can be determined. The related energy functionals are:

$$E_c(x_0) = \inf_{\substack{u \in L^2_\omega[0,T],\, T>0 \\ (T, x_0, u) = 0}} \mathcal{E}\left(\int_0^T \|u(t)\|^2 dt\right) \quad (8)$$

$$E_o(x_0) = \mathcal{E}\left(\int_0^\infty \left\|y(t, x_0, 0)\right\|^2 dt\right) \quad (9)$$

In the sequel, the relation between gramians of the stochastic systems and the energy functionals are described.

Theorem 1. [32]: *Consider the controllability gramian P and the observability gramian Q which are defined in (6) and (7) for the stochastic system (1).*