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Probabilistic analysis and control of systems with uncertain parameters over non-hypercube domain

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A B S T R A C T

Generalized polynomial chaos expansion provides a computationally efficient way of quantifying the influence of stochastic parametric uncertainty on the states and outputs of a system. In this study, a polynomial chaos-based method was proposed for an analysis and design of control systems with parametric uncertainty over a non-hypercube support domain. In the proposed method, the polynomial chaos for the hypercube domain was extended to non-hypercube domains through proper parameterization to transform the non-hypercube domains to hypercube domains. Based on the proposed polynomial chaos framework, a constrained optimization problem minimizing the mean under the maximum allowable variance was formulated for a robust controller design of dynamic systems with the parametric uncertainties of the non-hypercube domain. Several numerical examples ranging from integer to fractional order systems were considered to validate the proposed method. The proposed method provided superior control performance by avoiding the over-bounds from a hypercube assumption in a computationally efficient manner. From the simulation examples, the computation time by gPC analysis was approximately 10–100 times lower than the traditional approach.

1. Introduction

Modeling inaccuracies or uncertainties are inevitable in practice. The deterministic worst case setting is the most popular way of considering parametric uncertainty in control theory [1–3]. One of the drawbacks of this worst case-based method is its computational complexity, in that it often becomes computational intractable for general uncertainty structures [4]. To avoid this problem, the uncertainties are normally assumed to be of an interval or hypercube type in the worst case approaches. On the other hand, when the uncertainties are not of the hypercube type, they need to be over bounded by a hypercube, which can lead to an excessively conservative design. Therefore, a probabilistic approach that can handle the actual type of uncertainties should be considered for a proper controller design.

The Monte-Carlo (MC) method is a representative traditional probabilistic approach for the analysis and control of uncertain systems [4–6]. The brute-force implementation of the MC method involves first generating an ensemble of random realizations with each parameter drawn randomly from its uncertainty distribution. Solvers are then applied to each member to obtain an ensemble of results. The ensemble of results is then post-processed to estimate the relevant statistical properties, such as the mean, standard deviation, and density function. The stability and robustness of the system against the uncertainties can be inferred from these statistical properties. The estimation of the mean converges with the inverse square root of the number of runs, which makes MC methods computationally expensive. The high computational cost of MC methods has motivated the development of computationally efficient methods for uncertainty propagation and quantification that replaces or accelerates them, such as Quasi Monte Carlo (QMC) methods [4,7,8] and generalized polynomial chaos methods (gPC) [9–12].

Many studies have considered probabilistic approach for robust controller design [4,13–15]. These studies, however, focused primarily on MC/QMC methods rather than the gPC method. The most popular approach is to consider linear systems with stochastic additive Gaussian input [13]. For a linear time invariant system, the states are also Gaussian. Therefore, the predictive control problem is formulated as a standard chance constraint problem. When parametric uncertainties are involved, as reported in reference [14], the scenario approach is suggested for robust model predictive control. In reference [15], the MC method was used to design a full state feedback controller in mini unmanned aerial vehicles. Please refer to [16] for a recent update of the probabilistic methods for control system design.

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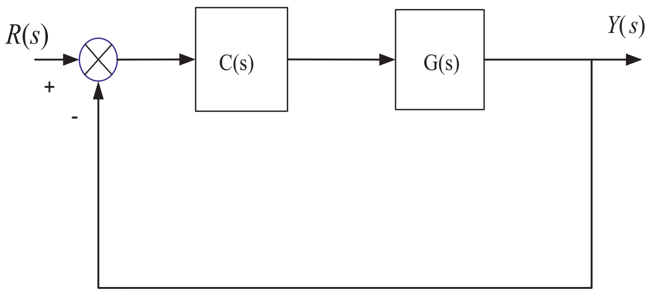

Fig. 1. Closed-loop control system.

This study considered generalized polynomial chaos (gPC) expansions as a functional surrogate model of a system model in the presence of uncertainty of the non-hypercube type. Although uncertainty propagation and quantification using gPC expansions have been studied extensively, the application of gPC to probabilistic robust control is relatively new [17,18], and unavailable to systems with uncertainty of the non-hypercube domain.

In this study, a polynomial chaos-based method was proposed for probabilistic analysis and robust controller design for systems with parametric uncertainty over the non-hypercube support domain.

This paper is organized as follows. Section 2 introduces the theory of gPC methods for uncertainty with a hypercube domain. Section 3 derives several parameterizations for an analysis of a non-hypercube domain. In Section 4, the gPC based method for the fraction order controller design is proposed. Section 5 includes several examples of probabilistic analysis and controller design of the systems with non-hypercube uncertainty.

2. Probabilistic analysis using polynomial chaos theory for hypercube domain

Consider a closed loop control system in Fig. 1 with a plant and a controller:

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu};$$
$$G(s) = \frac{b_m s^{\beta_m} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \ldots + a_0 s^{\alpha_0}} e^{-Ls} \tag{1}$$

where the vector of its parameter $\xi = (a_1, \ldots, a_n, b_1, \ldots, b_m, L) = (\xi_1, \xi_2, \ldots, \xi_N)$ is a random vector of mutually independent uniform random components with probability density functions of $\rho_i(\xi_i) : \Gamma_i \to \mathbb{R}^+$. Therefore, the joint probability density of the random vector, ξ, is $\boldsymbol{\rho} = \prod_{i=1}^{N} \rho_i$, and the support of ξ is $\boldsymbol{\Gamma} \equiv \prod_{i=1}^{N} \Gamma_i \in \mathbb{R}^N$. The set of one-dimensional orthonormal polynomials, $\{\phi_i(\xi_i)_{m=0}^{d_i}\}$, can be defined in finite dimension space, Γ_i, with respect to the weight, $\rho_i(\xi)$. Based on a one-dimensional set of polynomials, an N-variate orthonormal set can be constructed with P total degrees in the space, $\boldsymbol{\Gamma}$, using the tensor product of the one-dimensional polynomials that satisfies:

$$\int_{\Gamma} \Phi_m(\xi)\Phi_n(\xi)\rho(\xi)d\xi = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \tag{2}$$

Considering a response function of the system output $f(y(t, \xi))$ with the statistics (e.g. mean and variance) of interest, the N-variate Pth order approximation of the response function can be constructed as follows:

$$f_N{}^P(y(t,\xi)) = \sum_{i=1}^{M} \widehat{f}_m(t)\Phi_m(\xi);$$
$$M + 1 = \begin{pmatrix} N+P \\ N \end{pmatrix} = \frac{(N+P)!}{N!P!} \tag{3}$$

where P is the order of polynomial chaos, and $\widehat{f}_m$ is the coefficient of the gPC expansion that satisfies (2) as follows:

$$\widehat{f}_m = \mathbf{E}[\Phi_m f(y)] = \int_{\Gamma} f(y)\Phi_m(\xi)\rho(\xi)d\xi \tag{4}$$

where $\mathbf{E}[]$ denotes the expectation operator.

The probabilistic collocation approach [9] was used to obtain the gPC coefficients of the response function because of its simplicity. The algorithm is expressed briefly as follows:

- Choose a collocation set, $\{\xi_i^{(m)}, w^{(m)}\}_{m=1}^{qi}$ for each random component, ξ_i, for every direction $i = 1, \ldots, N$, and construct a one-dimensional integration rule,

$$Q_{q_i}^{(i)}[g] = \sum_{j=1}^{q_i} g(\xi_i^{(j)})w_i^{(j)} \tag{5}$$

where Q[ ] denotes the quadrature approximation of the univariate integration. A Gaussian quadrature [11] is normally used as a one-dimensional integral rule in classical spectral methods, such as the deterministic equivalent modeling method (DEMM).

- Obtain an N-dimensional integration rule by the tensorization of the one-dimensional integral rule:

$$\ell^Q[g] = (Q_{q_1}{}^{(1)} \otimes \ldots \otimes Q_{q_N}{}^{(N)})[g]$$
$$= \sum_{j_1=1}^{q_1} \ldots \sum_{j_N=1}^{q_N} g(\xi_1^{(j_1)}, \ldots, \xi_N^{(j_N)})(w_1{}^{(j_1)} \otimes \ldots \otimes w_N{}^{(j_N)}) \simeq \int_{\Gamma} g(\xi)\rho(\xi)d\xi \tag{6}$$

where $\otimes$ and $\ell^Q[]$ denote the tensor product and the multivariate quadrature (cubature) approximation, respectively.

- Approximate the gPC coefficients in (4) using the numerical integration rule in (6).

$$\widehat{f}_j = \ell^Q[f(y,\xi)\Phi(\xi)\boldsymbol{\rho}] = \sum_{m=1}^{Q} f(\xi^{(m)})\Phi_j(\xi^{(m)})w^{(m)} \, for \, j = 1, \ldots, M \tag{7}$$

where $\widehat{f}$ is the numerical approximation of $\widehat{f}$ using cubature.

- Construct an N-variate Pth order gPC approximation of the response function in the form, $\tilde{f}_N^P = \sum_{j=1}^{M} \widehat{f}_j \Phi_j(\xi)$.

Once all the gPC coefficients have been evaluated, a post-processing procedure is then carried out to obtain the statistics of the response function, $f(y(t, \xi))$.

The mean of the response function is the first expansion coefficient,

$$\mathbf{E}[\tilde{f}_N^P] = \mu_f = \int_{\Gamma} \tilde{f}_N^P \rho(\xi)\,d\xi = \int_{\Gamma} \left[\sum_{j=1}^{M} \widehat{f}_j(\xi)\Phi_j(\xi) \right] \rho(\xi)\,d\xi = \widehat{f}_1 \tag{8}$$