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Realization issues, tuning, and testing of a distributed predictive control algorithm

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ABSTRACT

A non-iterative, non-cooperative distributed state-feedback control algorithm based on neighbor-to-neighbor communication, named distributed predictive control (DPC), has been recently proposed in the literature for constrained linear discrete-time systems, see [15,14,2,4]. The theoretical properties of DPC, such as convergence and stability, its extensions to the output feedback and tracking problems, and applications to simulated plants have been investigated in these papers. However, for a practical use of DPC some realization issues are still open, such as the automatic selection of some tuning parameters, the initialization of the algorithm, or its response to unexpected disturbances which could lead to the lack of the recursive feasibility, a fundamental property for any model predictive control (MPC) technique.

This paper presents novel solutions to all these issues, with the goal to make DPC attractive for industrial and practical applications. Three realistic simulation examples are also discussed to evaluate the proposed numerical algorithms and to compare the performances of DPC to those of a standard centralized MPC algorithm.

1. Introduction

Due to the growing complexity of process plants and to the increasing number of networks of systems, in the last decades researchers have been putting huge efforts in the field of decentralized and distributed control [26,19]. Distributed solutions seem to be very promising with respect to decentralized schemes, because they allow one to take advantage of information transmission between the local controllers, see e.g. [18], and do not require the computational and communication loads of centralized solutions. However, distributed techniques are characterized by an intrinsically higher degree of complexity in the design phase with respect to centralized controllers. This could represent a great obstacle to their diffusion in the industrial world, and motivates the development of many innovative distributed model predictive control (MPC) algorithms for large-scale systems, see the survey papers [25,6] and the book [22], where the most recent and popular algorithms have been collected and described.

According to the classification of [25], a new non-iterative, non-cooperative approach based on neighbor-to-neighbor communication, called distributed predictive control (DPC), has been described in [15,14,2,4], where its convergence and stability properties have also been extensively analyzed. However, for a practical application of DPC, a number of issues concerning its realization and tuning have still to be solved and its performances must be assessed in realistic simulation scenarios. For these reasons, the aim of this paper is to consider and provide easy solutions to the main realization issues related to DPC (and similar distributed MPC methods), i.e. the use of a discretization method preserving the sparsity of the original continuous-time system, the computation of the required invariant sets, and the definition, both in the off-line and in the on-line phases, of the reference trajectories to be followed by the state and control variables. The proposed algorithms are then used for the realization and tuning of DPC applied to three realistic simulation problems. Specifically, the continuous-time models of the temperature dynamics in a simple building, of the level in a four tank system, and of a flotation process are controlled with DPC and the obtained performances are compared to those of a centralized model predictive control (cMPC) algorithm.

The paper is organized as follows: in Section 2 the DPC algorithm is summarized, while in Section 3.1 the discretization method, preserving the sparsity of the underlying continuous-time system, called mE-ZOH and originally presented in [7,8,12], is illustrated. Simplified procedures for computing the RPI sets are presented in Section 3.3. In Section 3.4, two techniques for the distributed design of the reference trajectories are proposed, while Section 4 contains the considered simulation examples. Some conclusions are drawn in Section 5.

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Notation. A matrix is Schur stable if all its eigenvalues lie in the interior of the unit circle. The short-hand $\mathbf{v} = (v_1, \ldots, v_s)$ denotes a column vector with s (not necessarily scalar) components $v_1, \ldots, v_s$. The symbol $\oplus$ denotes the Minkowski sum, namely $C = A \oplus B$ if and only if $C = \{c : c = a + b, \text{ forall } a \in A,\ b \in B\}$, while $\oplus_{i=1}^{M} A_i = A_1 \oplus \ldots \oplus A_M$. The Pontryagin difference is defined using the symbol $\ominus$, i.e. $C = A \ominus B$ if and only if $C = \{c : c + b \in A, \text{ forall } b \in B\}$. For a discrete-time signal s_t and $a, b \in \mathbb{N}$, $a \le b$, we denote $(s_a, s_{a+1}, \ldots, s_b)$ with $s_{[a:b]}$. Given a generic compact set $\mathcal{L}$, $\mathcal{H} = \text{box}(\mathcal{L})$ is the smallest hyper-rectangle containing $\mathcal{L}$ with faces perpendicular to the cartesian axis. Finally, $\|\mathcal{L}\|_\infty = \max_{l \in \mathcal{L}} \|l\|_\infty$.

2. The basic distributed predictive control algorithm

In this section, the distributed predictive control (DPC) algorithm first presented in [15] and further developed in [4] is briefly described. Let us assume that the system is constituted by M linear, discrete-time, non-overlapping subsystems, dynamically coupled through states and inputs, and subject to state and control constraints. For each subsystem $\mathcal{S}_i$, the dynamics is given by

$$\mathbf{x}_{k+1}^i = \mathbf{A}_{ii}\mathbf{x}_k^i + \mathbf{B}_{ii}\mathbf{u}_k^i + \sum_{j=1, j\neq i}^{M} \{\mathbf{A}_{ij}\mathbf{x}_k^j + \mathbf{B}_{ij}\mathbf{u}_k^j\} + \mathbf{d}_k^i \tag{1}$$

where $\mathbf{x}_k^i \in \mathcal{X}_i \subseteq \mathcal{R}^{n_i}$ and $\mathbf{u}_k^i \in \mathcal{U}_i \subseteq \mathcal{R}^{m_i}$ are the state and input vectors of the ith subsystem $\mathcal{S}_i$ ($i = 1, \ldots, M$), $\mathbf{d}_k^i \in \mathcal{D}_i \subset \mathcal{R}^{n_i}$ is an unknown bounded disturbance and the sets $\mathcal{X}_i$, $\mathcal{U}_i$ and $\mathcal{D}_i$ are convex neighborhoods of the origin. The subsystem $\mathcal{S}_j$ is said to be a *neighbor* of the subsystem $\mathcal{S}_i$ if and only if $\mathbf{A}_{ij} \neq 0$ and/or $\mathbf{B}_{ij} \neq 0$, i.e., if and only if the states x_j and/or inputs u_j of $\mathcal{S}_j$ influence the dynamics of $\mathcal{S}_i$. The symbol $\mathcal{N}_i$ denotes the set of neighbors of $\mathcal{S}_i$ (which excludes i).

Letting $\mathbf{x}_k = (\mathbf{x}_k^1, \ldots, \mathbf{x}_k^M)$, $\mathbf{u}_k = (\mathbf{u}_k^1, \ldots, \mathbf{u}_k^M)$ and $\mathbf{d}_k = (\mathbf{d}_k^1, \ldots, \mathbf{d}_k^M)$, the overall collective system can be written as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{d}_k \tag{2}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ have block entries $\mathbf{A}_{ij}$ and $\mathbf{B}_{ij}$ respectively, $\mathbf{x} \in \mathcal{X} = \prod_{i=1}^{M} \mathcal{X}_i \subseteq \mathcal{R}^n$, $n = \sum_{i=1}^{M} n_i$, $\mathbf{u} \in \mathcal{U} = \prod_{i=1}^{M} \mathcal{U}_i \subseteq \mathcal{R}^m$, $m = \sum_{i=1}^{M} m_i$, $\mathbf{d} \in \mathcal{D} = \prod_{i=1}^{M} \mathcal{D}_i \subset \mathcal{R}^n$, and $\mathcal{X}$, $\mathcal{U}$ are convex by convexity of $\mathcal{X}_i$ and $\mathcal{U}_i$, respectively.

Remark 1. System 2 can be seen as the state-space representation of a discrete-time empirical model obtained from data through identification procedures, for instance by means of impulse or step response experiments, or it can be computed as the the linearization and discretization of a continuous-time first principle model. In the latter case, the discretization procedure must guarantee to maintain the sparsity of the original continuous-time model, i.e., the mutual influences among the subsystems. This issue is discussed in the following Section 3.1.

The following assumption on decentralized stabilizability is needed.

Assumption 1. There exists a block diagonal matrix $\mathbf{K} = \text{diag}(\mathbf{K}_1, \ldots, \mathbf{K}_M)$, with $\mathbf{K}_i \in R^{m_i \times n_i}$, $i = 1, \ldots, M$ such that: (i) $\mathbf{A} + \mathbf{B}\mathbf{K}$ is Schur, (ii) $\mathbf{F}_{ii} = (\mathbf{A}_{ii} + \mathbf{B}_{ii}\mathbf{K}_i)$ is Schur, $i = 1, \ldots, M$.

At any time instant k, each subsystem $\mathcal{S}_i$ transmits to its neighbors its future state and input reference trajectories (to be later specified) defined over the prediction horizon N, and called $\tilde{\mathrm{x}}_{k+\nu}^i$ and $\tilde{\mathrm{u}}_{k+\nu}^i$, $\nu = 0, \ldots, N-1$, respectively. These trajectories coincide with the *assumed trajectories* introduced in [10]. By adding suitable constraints to its MPC formulation, $\mathcal{S}_i$ is able to guarantee that, for all $k \ge 0$, its real trajectories lie in specified time invariant neighborhoods of $\tilde{\mathrm{x}}^i$ and $\tilde{\mathrm{u}}^i$, i.e., $\mathbf{x}_k^i \in \tilde{\mathrm{x}}_k^i \oplus \mathcal{E}_i$ and $\mathbf{u}_k^i \in \tilde{\mathrm{u}}_k^i \oplus \mathcal{E}_i^{\mathcal{U}}$, where $0 \in \mathcal{E}_i$ and $0 \in \mathcal{E}_i^{\mathcal{U}}$. In this way, the dynamics (1) of $\mathcal{S}_i$ can be written as

$$\mathbf{x}_{k+1}^i = \mathbf{A}_{ii}\mathbf{x}_k^i + \mathbf{B}_{ii}\mathbf{u}_k^i + \sum_{j\in\mathcal{N}_i} \{\mathbf{A}_{ij}\tilde{\mathbf{x}}_k^j + \mathbf{B}_{ij}\tilde{\mathbf{u}}_k^j\} + \mathbf{w}_k^i \tag{3}$$

where

$$\mathbf{w}_k^i = \sum_{j\in\mathcal{N}_i} \{\mathbf{A}_{ij}(\mathbf{x}_k^j - \tilde{\mathbf{x}}_k^j) + \mathbf{B}_{ij}(\mathbf{u}_k^j - \tilde{\mathbf{u}}_k^j)\} + \mathbf{d}_k^i \in \mathcal{W}_i$$

and $\mathcal{W}_i = \oplus_{j\in\mathcal{N}_i}\{\mathbf{A}_{ij}\mathcal{E}_j \oplus \mathbf{B}_{ij}\mathcal{E}_j^{\mathcal{U}}\} \oplus \mathcal{D}_i$.

Each subsystem, using the algorithm proposed in [20], solves a robust MPC problem considering that its dynamics is given by (3), where the term $\sum_{j\in\mathcal{N}_i}(\mathbf{A}_{ij}\tilde{\mathbf{x}}_{k+\nu}^j + \mathbf{B}_{ij}\tilde{\mathbf{u}}_{k+\nu}^j)$ represents an input known in advance over the prediction horizon $\nu = 0, \ldots, N-1$, to be suitably compensated, and $\mathbf{w}_k^i$ is a bounded disturbance to be rejected.

Similarly to [20], a nominal model of subsystem $\mathcal{S}_i$ is associated to Eq. (3)

$$\mathrm{x}_{k+1}^i = \mathbf{A}_{ii}\mathrm{x}_k^i + \mathbf{B}_{ii}\mathrm{u}_k^i + \sum_{j\in\mathcal{N}_i} \{\mathbf{A}_{ij}\tilde{\mathbf{x}}_k^j + \mathbf{B}_{ij}\tilde{\mathbf{u}}_k^j\} \tag{4}$$

while the control law to be used for $\mathcal{S}_i$ is

$$\mathbf{u}_k^i = \mathrm{u}_k^i + \mathbf{K}_i(\mathbf{x}_k^i - \mathrm{x}_k^i) \tag{5}$$

where $\mathbf{K}_i$ must be chosen to satisfy Assumption 1.

Letting $\mathbf{z}_k^i = \mathbf{x}_k^i - \mathrm{x}_k^i$, in view of (3)–(5) one has

$$\mathbf{z}_{k+1}^i = \mathbf{F}_{ii}\mathbf{z}_k^i + \mathbf{w}_k^i \tag{6}$$

where $\mathbf{w}_k^i \in \mathcal{W}_i$. Since $\mathcal{W}_i$ is bounded and $\mathbf{F}_{ii}$ is Schur, there exists a robust positively invariant (RPI) set $\mathcal{Z}_i$ for (6) such that, for all $\mathbf{z}_k^i \in \mathcal{Z}_i$, then $\mathbf{z}_{k+1}^i \in \mathcal{Z}_i$. Given $\mathcal{Z}_i$ define, if possible, two sets, neighborhoods of the origin, $\Delta\mathcal{E}_i$ and $\Delta\mathcal{U}_i$, $i = 1, \ldots, M$ such that $\Delta\mathcal{E}_i \oplus \mathcal{Z}_i \subseteq \mathcal{E}_i$ and $\Delta\mathcal{U}_i \oplus K_i\mathcal{Z}_i \subseteq \mathcal{E}_i^{\mathcal{U}}$, respectively.

At any time instant k each subsystem $\mathcal{S}_i$ solves the following i-DPC problem.

$$\min_{\mathrm{x}_k^i, \mathrm{u}_{[k:k+N-1]}^i} \quad V_i^N = \sum_{\nu=0}^{N-1} (\|\mathrm{x}_{k+\nu}^i\|_{\mathbf{Q}_i^o}^2 + \|\mathrm{u}_{k+\nu}^i\|_{\mathbf{R}_i^o}^2) + \|\mathrm{x}_{k+N}^i\|_{\mathbf{P}_i^o}^2 \tag{7}$$

subject to (4),

$$\mathbf{x}_k^i - \mathrm{x}_k^i \in \mathcal{Z}_i \tag{8}$$

and, for $\nu = 0, \ldots, N-1$

$$\mathrm{x}_{k+\nu}^i - \tilde{\mathrm{x}}_{k+\nu}^i \in \Delta\mathcal{E}_i \tag{9}$$

$$\mathrm{u}_{k+\nu}^i - \tilde{\mathrm{u}}_{k+\nu}^i \in \Delta\mathcal{U}_i \tag{10}$$

$$\mathrm{x}_{k+\nu}^i \in \hat{\mathcal{X}}_i \subseteq \mathcal{X}_i \ominus \mathcal{Z}_i \tag{11}$$

$$\mathrm{u}_{k+\nu}^i \in \hat{\mathcal{U}}_i \subseteq \mathcal{U}_i \ominus \mathbf{K}_i\mathcal{Z}_i \tag{12}$$

and to the terminal constraint

$$\mathrm{x}_{k+N}^i \in \hat{\mathcal{X}}_i^F \tag{13}$$

The choice of the positive definite matrices $\mathbf{Q}_i^o$, $\mathbf{R}_i^o$, and $\mathbf{P}_i^o$ in (7) is discussed in Section 3.2 to guarantee stability and convergence, while $\hat{\mathcal{X}}_i^F$ in (13) is a nominal terminal set which must be chosen to satisfy the following assumption.

Assumption 2. Letting $\hat{\mathcal{X}} = \prod_{i=1}^{M} \hat{\mathcal{X}}_i, \hat{\mathcal{U}} = \prod_{i=1}^{M} \hat{\mathcal{U}}_i$ and $\hat{\mathcal{X}}^F = \prod_{i=1}^{M} \hat{\mathcal{X}}_i^F$, it holds that: