Journal of Process Control 24 (2014) 326–335

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Piece-wise constant predictive feedback control of nonlinear systems

Antonios Armaou [a,*], Armin Ataei [b]

[a] Department of Chemical Engineering, The Pennsylvania State University, University Park, PA 16802, USA
[b] Department of Electrical and Computer Engineering, Boston University, Boston, MA 02215, USA

ARTICLE INFO

Article history:
Received 1 July 2013
Received in revised form 13 January 2014
Accepted 3 February 2014

Keywords:
Process control
Receding horizon control
Optimal control
Carleman linearization
Dynamic optimization
BCMPC

ABSTRACT

We investigate the problem of receding horizon control for a class of nonlinear processes. A computationally efficient method is developed to identify the optimal control action with respect to predefined performance criteria. Using Carleman linearization and assuming piece-wise constant control action, the state vector is discretized explicitly in time. The optimal control problem is then reformulated as a nonlinear optimization problem and is efficiently solved using analytically computed sensitivity functions and standard gradient-based algorithms.

1. Introduction

Ever since the 1940s with the development of a general theory on controller design based on transfer function models, the idea of employing a model to assess system behavior and suppress deviations from the desired objective trajectories has been central to control systems theory [1]. Following the development of a solid foundation to linear systems theory based on frequency and state space methods, research focused on the development of a general theory on nonlinear systems design. The pursuit of this chimera has led to significant results on linear, nonlinear and stochastic control [2]. These results usually assume that the operator has unlimited power over the system. However, when considering the control problem of industrial processes, that is hardly the case [3].

In industrial practice, one is often faced with systems under various state, input and performance constraints [3–6]. A proper control design method is therefore one that can explicitly addresses these process constraints. Model predictive control (MPC), also known as receding horizon control, is one such powerful tool for handling these process constraints within an optimal control setting. At each sampling time, a control action is calculated by solving a finite-horizon open-loop optimization problem. The control action corresponding to the first sampling time is then implemented and the problem is solved again at the next sampling time with an extended horizon (hence the name receding horizon control). Since computations are repeated at each sampling time, the control action can potentially be altered on-line to suppress external disturbances and tolerate model inaccuracies. Extensive reviews on various MPC formulations along with their corresponding control-relevant issues such as closed-loop stability, performance and constraint satisfaction can be found in [7–10]. Other approaches on constrained control include anti-windup schemes [11] and Lyapunov designs [12].

As the MPC method is based on solving an underlying optimization problem, the stability guarantees are naturally linked to feasibility of such optimization problem. In this context, an important issue is the effect of initial condition on the feasibility of the optimization problem. To address this issue, [13] proposed Lyapunov-based explicitly defined regions of attraction for the closed loop system. The resulting Lyapunov-based controllers, however, are not guaranteed to be optimal with respect to an arbitrary performance criteria and do not admit performance criteria in their design. Some of these concerns have been addressed in [14–16] by introducing a Lyapunov-based MPC formulation that guarantees stability from an explicitly characterized set of initial conditions in presence of state and input constraints. The MPC formulation has also been extended for applications involving switched nonlinear systems and hybrid systems [17,14,18]. The control issue for systems exhibiting two time-scale behavior has also been addressed via model reduction and a dual controller design in [19,20]. More recently, research has focused on employing

* Corresponding author. Tel.: +1 814 8655316; fax: +1 814 8657846.
E-mail addresses: armaou@psu.edu, armaou@engr.psu.edu (A. Armaou), ataei@bu.edu (A. Ataei).

http://dx.doi.org/10.1016/j.jprocont.2014.02.002
0959-1524/

economic criteria to define MPC cost functions [21,22]. Portfolio optimization and index tracking problems have also been investigated in [23,24].

The main focus of this manuscript is to develop a computationally efficient method for solving an optimal control problem for a class of nonlinear systems. The optimal control problem is formulated as a receding control horizon one, thus it requires solving a dynamic optimization problem at each time step. Employing nonlinear transformations and assuming piece-wise constant control action, the dynamic optimization problem is reformulated as a nonlinear programming (NLP) problem with analytically computed sensitivity functions. This allows for the problem solution using standard, gradient-based, search algorithms. The proposed method lies at the interface between collocation and shooting methods. The system states are discretized explicitly in time and their sensitivity to the control action is analytically computed, reminiscent of collocation methods [25], while the states now enter the optimization problem explicitly as a nonlinear function of the control action and are eliminated from the equality constraints, thus reducing the number variables, evocative of shooting methods [26,27].

2. Process description and preliminaries

The proposed controller design method deals with the design of predictive controllers for bilinear and general nonlinear systems, subject to nonlinear constraints, employing piece-wise constant control actions.

The central objective of this work is to employ nonlinear mapping techniques, in conjunction with nonlinear temporal discretization (dynamic models which accurately describe the evolution of $x_i(t)$ based on information from microscopic simulations). Specifically, we focus on nonlinear models of the following form:

$$\dot{x} = f(x) + g(x)u = f(x) + \sum_{j=1}^{m} g_j(x)u_j(t), \quad x(0) = x_0 \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the vector of manipulated variables and $u_j(t)$, is the jth element of u. $f(x)$ is a nonlinear vector function of the state, and $g_j(x)$ is a nonlinear vector function, which accounts for the influence of the jth control actuator on the process. Without loss of generality, we assume that the target steady state of the system is the origin.

To facilitate the presentation, we will use the following notation in the remainder of the manuscript. The Kronecker product between matrices $A \in \mathbb{C}^{N\times M}$ and $B \in \mathbb{C}^{L\times K}$ can be defined as a matrix $C \in \mathbb{C}^{(NL)\times(MK)}$, where

$$C = A \otimes B \equiv \begin{bmatrix} a_{1,1}B & a_{1,2}B & \cdots & a_{1,M}B \\ a_{2,1}B & a_{2,2}B & \cdots & a_{2,M}B \\ \cdots & \cdots & \cdots & \cdots \\ a_{N,1}B & a_{N,2}B & \cdots & a_{N,M}B \end{bmatrix}.$$

We also define the kth order Kronecker product as $A^{[k]} = A^{[k-1]} \otimes A$, $A^{[1]} = A$ and $A^{[0]} = 1$. $I_n \in \mathbb{R}^{n\times n}$ is defined as the identity matrix of dimension n. We denote the Boxcar function with $\mathbf{B}_U(t; t_i, t_f) = U(H(t - t_i) - H(t_f - t))$ a pulse function, where U is the pulse amplitude, t_i is the initiation time and t_f the termination time (H denotes the standard Heaviside function).

3. Predictive feedback controller design methodology

We focus on the design of predictive output feedback controllers with piece-wise constant control action. The method hinges on the formulation of the dynamic optimization problem as a constrained nonlinear problem.

We initially formulate the state feedback control problem and present the appropriate formulations for bilinear systems, followed by a formulations for nonlinear systems and finally output feedback controller designs.

A finite-horizon optimal control problem can be formulated as one of the general form:

$$\begin{aligned} u^* = \arg\min_u & \int_{t=t_0}^{t_f} J(x, u)\,\mathrm{d}t \\ \text{s.t.} & \\ \dot{x} - f(x) - g(x)u &= 0, \\ x(t_0) &= x_0, \\ f^c(x, u) &\le 0, \end{aligned}$$

where f^c denotes inequality constraints, which may be imposed as a result of physical limitations (e.g. the available control action), economic and production restrictions or artificial ones (e.g. a bound on the Euclidean norm of x).

We focus our attention to problems where the control action that can be implemented involves the decision at discrete time instants of what the control action should be over a certain period of time. Function $u(t)$ then attains a piece-wise constant form with respect to time. Let us assume that a finite number of such control action decisions are to be taken. We denote the sequence of N decisions that are taken for manipulated variable u_j with $U_j \in \mathbb{R}^{1\times N}$. Similarly, we denote the control sequence of the manipulated variable vector u with U. Obviously $U \in \mathbb{R}^{m\times N}$. The problem can now be reformulated as

$$\begin{aligned} U^* = \arg\min_U & \int_{t=T_0}^{T_f} J(x, u)\,\mathrm{d}t \\ \text{s.t.} & \\ u_j(t) = \sum_{i=1}^{N} & \mathbf{B}_{U_{j,i}}(t; T_{i-1}; T_i), \quad \forall j = 1, \ldots, m \\ \dot{x} - f(x) - \sum_{j=1}^{m} & g(x)u_j(t) = 0, \\ x(t_0) &= x_0, \\ f^c(x, u) &\le 0 \end{aligned} \tag{2}$$

The decision variables are organized in the matrix form $U = [U_{j,i}]$, where i and j correspond to the ith decision for the jth manipulated variable. $(T_{i-1}, T_i]$ is the time period of the ith control action, T_0 is the initial time, and $T_f = T_N$ is the final time.

Remark 1. Some nonlinear model predictive control formulations assign a final time penalty or a final time constraint to the optimization problem. With slight abuse of notation, we assume that such a constraint is incorporated directly to either J or f^c, which are otherwise independent of time. Generally, Dirac, Heaviside and Boxcar type functions of time can be directly incorporated to the optimization problem with minor modifications to the mathematical expressions that follow.

Remark 2. In receding horizon control, it is customary to have different control and prediction horizons. The choice of these two horizons is important to obtain stable and robust controllers [7]. In the present problem, the choice of the control horizon lies in the choice of the time periods $\Delta T_i = T_i - T_{i-1}$, $\forall\, i = 1, \ldots, N-1$ (which are customarily chosen to be of equal value) and the choice of the prediction horizon lies in the choice of ΔT_N.