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Stability analysis and design of integrating unstable delay processes using the mirror-mapping technique

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ABSTRACT

Robustness analysis and design for the integrating unstable delay systems are discussed in this note. The Nyquist criteria have established the exact stability margin of the novel robust control scheme, which is meaningful in the process control practice. Comparing with the existing results, the control law is designed based on the delay-approximated model (using the all-pole Padé approximation). The unstable system was mirror mapped into a minimum-phase system and then the control law was derived by the closed-loop gain shaping algorithm (CGSA). In addition, a small constant δ was introduced in the algorithm to prevent the integral cancellation limitation, which is inherent in the CGSA. The proposed scheme obtains several advantages: a concise design procedure and easy to implementation due to the simple unit feedback structure. The comparative analysis with respect to recently successful works illustrates a substantial improvement in the performance-robustness tradeoff.

1. Introduction

Unstable system is frequently encountered in the practical control engineering. And most industrial processes, involving material or energy transportation, are characterized by the presence of time delay. Control of unstable plant coupled with time delay has been an active research field due to its theoretical difficulties and important applications [1], e.g. chemical reactors, distillation columns, unstable very large carriers, etc.

The integrating unstable delay plant belongs to the hardly controllable system, with regard to the presence of both integrating and unstable terms. In addition, the distinctiveness of non-minimum phase dynamic would blame the stability analysis and design for the quasi eigenpolynomial. Eventhough, several significant works have been presented in the current literatures [2–8]. Motivated by the convenient application in process control industry, the stabilization conditions of simple controllers are discussed in [2–5] by virtue of the Nyquist stability criteria, i.e. P/PI/PD/PID type control law. However, the problems of closed-loop control performance and robustness are not detailed in these works. Based on the internal model control (IMC) structure [9], a modified IMC is developed to design and tune the controllers with double two-degree-of-freedom (TDF) scheme for integrating and unstable delay systems [6]. Robustness measure have illustrated the improved robust performance of the double TDF scheme assuming that uncertainties are with a general formal description. In [10,7], using the spectral factorization techniques and the polynomial approach, a systematic design procedures were presented with the TDF control system structure (two feedback controller) to improve the acceptable performance. Recently, [8] presents a modified smith predictor (MSP) for stable, integrating and unstable processes. A key conclusion is obtained that the MSP is a PID controller in series with a second order filter, and the actual control parameters are optimized using the Particle Swarm Optimization. In addition, there also exists the other novel approaches for controlling the integrating and unstable systems, such as the eigenvalue-loci technique [11] and the enhanced cascade control scheme [12].

In these forementioned works, it is obvious to note that most of the reported processes are only with the unstable term or the integrating one. And for several cases the existed stable poles may ease the regulatory task by assigning the predominant pole of the closed-loop system. To the best of the authors' knowledge, only [8,7] obtain the valid results for the integrating unstable delay processes with the complex structures. Motivated by the above observation, the further effective stabilizing research is deserved for the integrating unstable delay process.

In recent years, authors of this note were devoted to developing new algorithms in controlling of unstable processes. In [13,14], mirror mapping method is first developed as a new tool for control design of the regular unstable system. The unstable poles (or zeros) are mirror mapped onto the left half plane so that the unstable

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plant is transformed into a minimum-phase system. It is noted that the largest singular value curves of unstable processes are identical before and after the mirror mapping operation. Therefore, the control law has equal robustness to both the unstable process and the mirror mapping process by applying the closed-loop gain shaping algorithm (CGSA) [15,16]. However, the mirror mapping technique could not be directly used for the pure unstable process. In [17], the control problem of the pure unstable system was solved by neglecting the time delay, and a component with zeros and poles in the left half plane was introduced to convert the pure unstable system into a regular one, which is difficult to choose. The result in [18] was to design a controller for unstable process with dual poles (i.e.$(s+\omega_1)(s-\omega_1)$) in the denominator, which had been used into the maglev train with satisfactory performances. And [19,20] solved the control problem of static unstable missile with dual poles and dual zeros (i.e. $[(s+\lambda_1)(s-\lambda_1)]/[(s+\omega_1)(s-\omega_1)]$) using the mirror mapping method.

In this note, a novel robust control design is derived for the integrating unstable delay process using the mirror mapping technique and CGSA. The time delay term is transformed as a stable component by means of the all-pole Padé approximation, and the Nyquist criteria establishes the exact stabilization of the closed-loop system.

2. Problem formation and preliminaries

In this note, one devotes to developing the stability analysis and effective design via the simple control structures, i.e. the unit output feedback scheme given in Fig. 1. r is the reference signal, d_1 is the load disturbance, d_2 is the output disturbance. The transfer function of unstable plus integrating time delay system has the form of (1). p is the Laplace operator of the transfer function, which is employed to distinguish the normalized one "s" in (2). Following [4], the normalization (2) is adopted to illustrate the stabilization problem with fewest possible parameters. The normalized plant and controller of interest are given in (3), where $L=\overline{\tau}_d/\overline{\tau}\geq 0$. Eq. (3) would also be used to derived the actual control law $\overline{C}(s)$ in the practical industry.

$$\overline{G}(p)=\frac{\overline{k}}{p(\overline{\tau}p-1)}e^{-\overline{\tau}_d p},\ \overline{k}>0,\ \overline{\tau}>0,\ \overline{\tau}_d\geq 0 \tag{1}$$

$$\begin{aligned}\overline{C}(p)\overline{G}(p) &= \overline{C}(p)\frac{\overline{k}}{p(\overline{\tau}p-1)}e^{-\overline{\tau}_d p},\\ (s=\overline{\tau}p) &\Rightarrow \overline{k}\overline{\tau}\overline{C}\left(\frac{s}{\overline{\tau}}\right)\frac{1}{s(s-1)}e^{-Ls}=C(s)G(s)\end{aligned} \tag{2}$$

$$G(s)=\frac{1}{s(s-1)}e^{-Ls},\ C(s)=\overline{k}\overline{\tau}\overline{C}\left(\frac{s}{\overline{\tau}}\right) \tag{3}$$

Using the 1st/2nd all-pole Padé approximation (4), the time delay term in (3) is approximated without introducing the unwanted positive poles or zeros. Whereas, the other Padé methods detailed in [21,22] would introduce the undesired positive pole (or zero) into the approximated system, e.g. $e^{-Ls}\approx(2-Ls)/(2+Ls)$, $e^{-Ls}\approx(1-0.6143Ls+0.1247L^2s^2/2)/(1+0.3866Ls)$. Finally, the corresponding approximated system models (5) are obtained for the following control design.

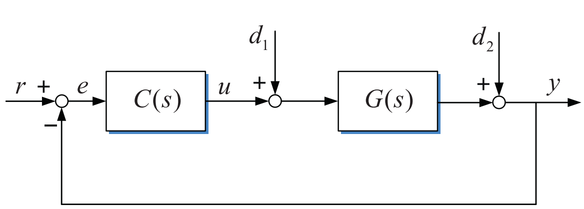

Fig. 1. The unit output feedback scheme.

$$\begin{aligned}&\text{1st order approximation,} e^{-Ls}\approx\frac{1}{Ls+1}\\ &\text{2nd order approximation,} e^{-Ls}\approx\frac{1}{((L^2s^2)/2)+Ls+1}\end{aligned} \tag{4}$$

$$\begin{aligned}G_{A1}(s)&=\frac{1}{s(s-1)(Ls+1)}\\ G_{A2}(s)&=\frac{1}{s(s-1)\left(((L^2s^2)/2)+Ls+1\right)}\end{aligned} \tag{5}$$

In order to use the Nyquist stability criteria, the open-loop transfer function $Q(s)$ should be expressed as the following (6).

$$Q(s)=C(s)G(s)=\frac{KN(s)}{s^{v}D(s)}e^{-Ls} \tag{6}$$

where K is the gain, $v\geq 0$ represents the type of the loop. $N(s)$, $D(s)$ are the corresponding numerator/denominator polynomials of s with $N(0)=D(0)=1$. Lemma 1 presents the Nyquist stability criterion applying to the open-loop transfer function (6). The Nyquist contour is with the symmetry property to the real axis. Therefore, subsequent analysis focuses on the positive frequency $\omega\in(0,+\infty)$.

Lemma 1. *Given the open-loop transfer function $Q(s)$ in (6) with P^{+} unstable poles, the closed-loop system is stable [3]*

(1) *if and only if the Nyquist plot of $Q(s)$ encircles the critical point $(-1,0)$ P^{+} times anticlockwise.*
(2) *only if $\lim_{\omega\to\infty}|Q(j\omega)|<1$.*
(3) *only if $K<-1$ when $P^{+}=1$, $v=0$, or $K<0$ when $P^{+}=1$, $v=1,2$.*
(4) *only if the polynomial, $H(s)=e^{-Ls}\frac{d^{m+1}}{ds^{m+1}}\left[s^{v}D(s)e^{Ls}\right]$, has all roots in the open left half plane, where m is the degree of $N(s)$.*

2.1. CGSA and the mirror-mapping technique

CGSA is a simplified H_∞ mixed sensitivity algorithm [15,16] by shaping directly the singular value curves of the sensitivity function $S(S=1/(1+GC))$ and the complementary sensitivity function $T(T=GC/(1+GC))$ shown in Fig. 2, and there exists the correlativity $T=I-S$ between S and T. According to the H_∞ robust control theory, the closed-loop frequency spectrum of a typical control system, i.e. T, has a low pass characteristics to guarantee the robust performance, and the largest singular value equals to unit one to follow

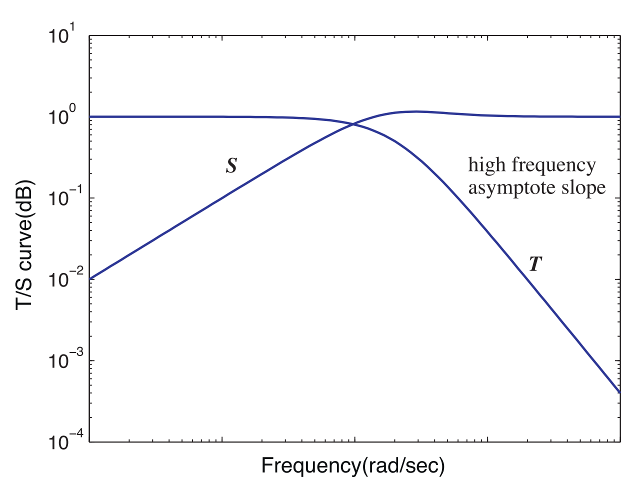

Fig. 2. Typical S&T singular value curves.