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Rapid distributed model predictive control design using singular value decomposition for linear systems

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ABSTRACT

The issue of model predictive control design of distribution systems using a popular singular value decomposition (SVD) technique is addressed. Namely, projection to a set of conjugate structure is dealt with in this paper. The structure of the resulting predictive model is decomposed into small sets of subsystems. The optimal inputs can be separately designed at each subsystem in parallel without any interaction problems. The optimal inputs can be directly obtained and the communication among the subsystems can be significantly reduced. In addition, the design of distribution model predictive control (DMPC) with constraints using the SVD framework is also presented. The unconstraint inputs are checked in parallel in the conjugate space. Without solving the QP problem of each subsystem, the suboptimal solution can be quickly obtained by selecting the bigger singular values and discarding the small singular values in the singular value space. The convergence condition of the proposed algorithm is also proved. Two case studies are used to illustrate the distribution control systems using the suggested approach. Comparisons between the centralized model predictive control method and the proposed DMPC method are carried out to show the advantages of the newly proposed method.

1. Introduction

Recently both the theoretical developments and the applications of distributed model predictive control (DMPC) have received great attention [1], because operators faced large scale industrial systems with the problems of safety, environmental sustainability and profitability. Typical examples of large-scale systems included chemical processes [2], electrical power networks [3], irrigation canals [4,5], temperature regulation [6,7], supply chain [8] and urban traffic networks [9]. However, in practical consideration, centralized model predictive control (MPC) of large scale systems with the interconnected network of individual processing units is not easy to implement. For control of large-scale systems, DMPC is a good choice because of the low computational load for each subsystems and the protection of the local information. Besides large-scale systems, DMPC can cope with interactions by sharing control inputs and states among subsystems. Unlike decentralized MPC [10], which requires control schemes depending on any centralized element only, DMPC considers the distribution nature of interactions among subsystems. Although DMPC can lower the dimension of the optimal control problem of each subsystem, it is time-consuming for the controllers to communicate with each other to send and receive information, such as control inputs and predicted states. Therefore, how to cut the communication burden without degrading performance is a critical issue. As presented by Necoara et al. [11], the distributed algorithm for solving the coupled constraint problem is a tradeoff among convergence speed, message passing amount, and distributed computation architecture. Some papers on DMPC can be referred to [1,12,13].

Distribution control systems can offer many advantages, including effective resource utilization, simple installation and maintenance, high flexibility, reliability and tele-operation. However, compared with the centralized MPC, in each subsystem of DMPC, the substantial decrease of the number of controllers, decision variables, state variables and measurements decrease the computational time significantly. In order to cope with constraints and interactions among subsystems, communication among subsystems is inevitable. Therefore, a good DMPC theme is able to deal with interactions with the minimum communication load while keeping a good control performance. Farina and Scattolini [14] considered a linear discrete-time system with coupled states and coupled inputs. The objective function for each subsystem with

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only local states and inputs was considered, and the impact of its neighbors was treated as bounded disturbance, but the algorithm is effective under the assumption of decentralized stabilizability. Maestre et al. [15] proposed a novel DMPC algorithm based on the game theory for a class of systems controlled by two agents. The proposed controller only needed two communication steps in order to obtain a cooperative solution. Later Maestre et al. [16] extended the work to the multi-agent negotiation case. The negotiation took place in which the agents make different proposals and only one of them was chosen following a social criterion. Another way to cut down the communication load is to consider neighbor-to-neighbor communication only. Zheng et al. [17] proposed a DMPC with each subsystem exchanging a reduced set of information with its neighbors. The optimization index of each local MPC considered not only its own performance but also the performance of its output neighbors. Each subsystem only needs to communicate with its neighbors via neighborhood optimization. Zhang et al. [18] also applied the method to cascade systems. The objective function of each subsystem only considered the object of its upstream and downstream. However, if each subsystem is highly coupled with all the other subsystems, the communication load still could not be reduced.

There is not much work on nonlinear distributed model predictive control. Liu et al. [2,19,20] applied Lyapunov-based model predictive control [21,22] to nonlinear distributed model predictive control. An additional constraint is added to the optimal control problem to force the decrease of the Lyapunov function. In [19], the communication at each sampling time is sequential between two distributed controllers. In [2], the method was extended to multi-controllers, the controllers can communicate with each other sequentially or iteratively. In [3], asynchronous and delayed measurements were added to that framwork.

To achieve better closed-loop control performance, some levels of communication may be established between different controllers. In [23], the communication load is cut down because the evaluations of the distributed controllers are triggered by the difference between the subsystem state measurements and the estimates of them. The idea of cooperative DMPC was first introduced [24]. In cooperative DMPC, at each iteration, each controller optimizes its own set of inputs, but the rest of the inputs of its neighbors are fixed to the last agreed values. Subsequently, the resulting optimal trajectories from each controller are transmitted among subsystems. Based on a weighted sum of the most recent optimal computed trajectories at the last sampling time, a final optimal trajectory is computed. However, it is unable to deal with coupled inputs constraint. Stewart et al. [25] modified cooperative DMPC to handle coupled inputs constraints for the state feedback and the output feedback problems. After that, Stewart et al. [26] proposed a method that the subsystems were grouped in hierarchy, so the communication among all the subsystems was not necessarily needed. It reduced the communication frequency between subsystems without new levels of coordinating controllers. The above design procedure, so called the Jacobi algorithm, is an iterative design related to parallel computing [27]. Because of the optimal input from each subsystem, the inputs would limit the convergence speed. Therefore, there is no guarantee of how the optimal solution can be approximated after a certain number of iterations.

The Jacobi method does not have a good mechanism to deal with constraints. In order to cope with constraints efficiently and achieve fast convergence, recent research work on DMPC schemes solved DMPC problems via dual decomposition and gradient ascent. Doan et al. [28] proposed a decomposition approach based on Fenchel's duality and Han's parallel method. Necoara et al. [29] proposed a dual-based decomposition method. It is a proximal center method that derives decomposition schemes for convex optimization problems with a separable structure. Necoara et al. [30] also applied the decomposition method [29] to DMPC. These algorithms divide the optimization procedure into two steps. The first step is to solve an unconstraint optimal control problem in parallel; the second, to update the Lagrange parameters and deal with constraints in a centralized manner. However, the centralized method does not fully use the distributed controller design; also, the implementation would be inflexible. Another way to deal with constraints is presented [31]. The constraints are allocated to each subsystem; then the Lagrange parameters and the control inputs should be coordinated and communicated. However, a convergence analysis must be carried out first before the implementation of the above method. This limits the applications of the method.

The large industrial processes are characterized by an interconnected network of the individual processing units, such as chemical reactors, distillation columns, heat exchangers and mixing tanks. Because of the presence of cycle loops and the transportation lag, the measurements of controlled process variables are expected to be strongly correlated. This means that the degree of interaction among the input and output variables is usually of high importance. In the past, several measures of interaction proposed with relative gain array (RGA) were widely used in process control [32,33]. The measure of the degree of interaction was useful for selecting the input–output pairings to build independent controller loops. However, when the RGA elements, corresponding to all input–output pairings, were substantially far from unity, there was still interaction from each control loop and it was difficult to achieve perfect decoupling. The singular value decomposition (SVD) of the process gain matrix was also proposed for decoupling [34]. The process was transformed by a post-multiplied matrix and a pre-multiplied matrix. The resulting transformed inputs and outputs were independently or completely decoupled. The advantages of SVD have been demonstrated in MPC [35]; the optimal inputs are easily obtained in the project space.

Unlike those DMPC, SVD decomposition can get the completely decoupled inputs. It can reduce the number of the input variables and the variables are orthogonal. It is worthy of applying the SVD decomposition strategy to the DMPC problem. In this research, we extend the concept to distributed MPC. SVD for DMPC (SVD-DMPC) is proposed to address problems, such as the low speed of the convergence in DMPC and the impractical centralized way of dealing with constraints. The conjugate input directions can be obtained before the on-line application; without the interaction problems, the optimal inputs can be separately designed at each subsystem in parallel. Under the parallel structure, the communication method among the subsystems is proposed to solve the optimization problem. To deal with constraint problems, without the needed computation load of the constraint QP problems, a simple design method in the conjugate input space is proposed. It keeps the conjugate inputs with the largest singular values while discarding in order the ones with the smallest singular values. The remainder of this paper is organized as follows. In Section 2, the optimal control problems of centralized MPC and Jacobi based DMPC (JDMPC) are reviewed. In Section 3, SVD-DMPC is proposed and the comparisons of the iterate trajectory of JDMPC with SVD-DMPC are included. Then the SVD-DMPC problem is extended to the control problems with constraints in Section 4. Without solving the centralized QP problem, the convergence of the proposed design strategy is also proved. The effectiveness of the proposed method is demonstrated through a simple two-input two-output mathematical problem and a simulation benchmark of the alkylation of the benzene process in Section 5. These examples investigate the performance of the proposed method and make a comparison with conventional algorithms (centralized MPC and JDMPC). Finally, some concluding remarks are made.