



A new exponentially weighted moving average sign chart using repetitive sampling



Muhammad Aslam^a, Muhammad Azam^{a,*}, Chi-Hyuck Jun^b

^a Department of Statistics, Forman Christian College University, Lahore, Pakistan

^b Department of Industrial and Management Engineering, POSTECH, Pohang 790-784, Republic of Korea

ARTICLE INFO

Article history:

Received 1 July 2013

Received in revised form 4 February 2014

Accepted 9 May 2014

Available online 2 June 2014

Keywords:

Binomial distribution

EWMA

Repetitive sampling

Sign statistic

Average run length

ABSTRACT

In this paper, a new nonparametric control chart based on the exponentially weighted moving average (EWMA) sign statistic is proposed using repetitive sampling. The control chart is proposed to effectively detect the process mean shift away from the target value without the distributional assumption on the quality characteristic. The proposed control chart is based on two pairs of upper and lower control limits having different control coefficients. The in-control and the out-of-control average run lengths of the proposed control chart are evaluated through the Monte Carlo simulation. The proposed control chart is shown to be more efficient than the existing EWMA sign control chart in terms of the average run length.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

The good quality of the product brings a good reputation to an industry in the global market competition. The improvement in the quality is one of the main aims of the industries of the world. In simple words, we can say that without the gradual improvement in the quality, the survival of an industry will not be guaranteed. The improvement in the quality is only possible if the industrialist pays full attention to the management from the raw material to the manufacturing process.

Control charts including X-bar, EWMA and CUSUM for variable data are playing an important role on the improvement in the quality of the products. These types of control charts provide the facility to monitor the process and quick indication when the process is going to be out-of-control. The Shewhart control chart is useful to detect relatively large shifts in the process. On the other hand, the EWMA and CUSUM control charts are powerful tools to monitor small shifts in the process. Usually, a control chart for variable data is designed by assuming that the quality characteristics follow the normal distribution. Control charts developed under this assumption may cause false alarms when the quality of interest follows a non-normal distribution. In this dimension, the

following studies for example are available in the literature. Ferrell [16] proposed the control charts for midranges and medians. Bakir and Reynolds [7] designed a non-parametric control chart using within group ranking. Amin et al. [4] designed a non-parametric control chart based on sign statistic. Chakraborti et al. [17] presented a review on non-parametric control charts. Altukife [2,3] proposed non-parametric control charts based on observations exceeding the grand mean and the sum of ranks, respectively. Bakir [5,6] proposed a distribution-free control chart using the signed ranks and sign rank-like statistics. Chakraborti and Eryilmaz [13] proposed the non-parametric Shewhart type sign rank control chart. Chakraborti and Graham [11] studied the non-parametric control charts. Chakraborti and Van der Wiel [12] proposed the control chart using the Mann–Whitney statistic. Das and Bhattacharya [14] proposed a non-parametric control chart to control variability. Borror and Keats [9] proposed the control chart for time between event data. Recently, Yang et al. [22] developed a new control chart using the EWMA sign statistic. More details about control chart can be seen in FallahNezhad and Niaki [15], Gopalakrishnan et al. [17], Lucas and Saccucci [18] and Mosteller and Youtz [20].

By exploring the literature of control charts, we note that the control charts based on normal or non-normal distributions are utilizing single sampling or double sampling schemes at each subgroup. One of the important sampling schemes that have been widely used in the area of quality control and acceptance sampling plans is repetitive sampling, which is originally proposed by Sherman [21]. The operational procedure of this sampling is quite

* Corresponding author. Tel.: +92 3218830975.

E-mail addresses: aslam_ravian@hotmail.com (M. Aslam), mazam72@yahoo.com (M. Azam), chjun@postech.ac.kr (C.-H. Jun).

similar to the sequential sampling. But, the repetitive sampling is more efficient than the single and double sampling schemes. On the other hand, this sampling is quite easy to apply as compared to the sequential sampling. Balamurali and Jun [8] developed the variable acceptance sampling plans using repetitive sampling and proved that the repetitive sampling is more efficient in reduction of the average sample number as compared to the single and double sampling.

As mentioned earlier, no attention has been paid to propose the control chart using the repetitive sampling. Here, in this paper, we will introduce a non-parametric EWMA sign control chart using repetitive sampling with the expectation that the proposed control chart is more efficient than the control chart proposed by Yang et al. [22] based on single sampling. The rest of the paper is set as follows: first, background of EWMA sign statistic is given in Section 2. The proposed control chart is introduced in Section 3 and its advantage is described in Section 4. Section 6 concludes this paper.

2. Background of EWMA sign statistic

In this section, we will provide some background of the EWMA sign statistic in Yang et al. [22]. Let X be the quality characteristic of interest with target value T . Let also $Y=(X-T)$ be the deviation of the process from the target value. Then, $p=P(Y>0)$ is called the process proportion. When $p=0.5$, the process is regarded to be in control and when $p=p_1 \neq 0.5$, the process is regarded to be out-of-control. For the monitoring purpose, a random sample of size n , X_1, X_2, \dots, X_n , is selected at each subgroup from the process. Define

$$I_j = \begin{cases} 1, & \text{if } Y_j = X_j - T > 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } j = 1, 2, \dots, n$$

Let M be the total number of values for $Y_j > 0$. That is,

$$M = \sum_{j=1}^n I_j$$

Then, M follows the binomial distribution with parameters n and $p=0.5$ for the in-control process. If M_i is the i th sequentially recorded value of such M , then the EWMA sign statistic is given as follows:

$$EWMA_{M_i} = \lambda M_i + (1 - \lambda)EWMA_{M_{i-1}},$$

where λ is the smoothing constant between 0 and 1. For more details, reader may refer to Yang et al. [22]. Abbasi [1] corrected the mistake in variance formula given by Yang et al. [22]. According to Abbasi [1], the mean and the variance are given by, when the process is in control.

$$E(EWMA_{M_i}) = n/2 \text{ and}$$

$$\text{Var}(EWMA_{M_i}) = \frac{\lambda}{2 - \lambda} \left(\frac{n}{4} \right)$$

3. Proposed EWMA sign chart

As mentioned in Introduction, the use of repetitive sampling is popular in lot acceptance sampling plans. We try to incorporate the idea of repetitive sampling in a non-parametric control chart with the expectation of detecting the process out-of-control more quickly because the decision is postponed by gathering more data with repetitive sampling. In other words, the decision (whether the process is in control or out of control) will be made based on a single sample when it is obvious but resampling will be performed

if it is dubious. We propose the following EWMA sign control chart having double control limits based on repetitive sampling:

Step 1: Take a sample of size n for the i -th subgroup. Calculate the EWMA sign statistic as follows:

$$EWMA_{M_i} = \lambda M_i + (1 - \lambda)EWMA_{M_{i-1}},$$

Step 2: Declare the process as out-of-control if $EWMA_{M_i} \geq UCL_1$ or $EWMA_{M_i} \leq LCL_1$. Declare in-control if $LCL_2 \leq EWMA_{M_i} \leq UCL_2$. Otherwise, go to Step 1 for resampling.

The operational procedure of the proposed chart is based on four limits LCL_1, LCL_2, UCL_1 and UCL_2 . The two upper control limits and the two lower control limits of the proposed chart are given as follows:

$$UCL_1 = \frac{n}{2} + k_1 \sqrt{\frac{\lambda}{2 - \lambda} \left(\frac{n}{4} \right)}, \quad LCL_1 = \frac{n}{2} - k_1 \sqrt{\frac{\lambda}{2 - \lambda} \left(\frac{n}{4} \right)} \quad (1a)$$

$$UCL_2 = \frac{n}{2} + k_2 \sqrt{\frac{\lambda}{2 - \lambda} \left(\frac{n}{4} \right)}, \quad LCL_2 = \frac{n}{2} - k_2 \sqrt{\frac{\lambda}{2 - \lambda} \left(\frac{n}{4} \right)} \quad (1b)$$

In Eq. (1), k_1 and k_2 are the control coefficients ($k_1 \geq k_2$) to be determined. When $k_1 = k_2$, the proposed control chart becomes the control chart proposed by Yang et al. [22]. So, the proposed control chart is an extension to the control chart proposed by Yang et al. [22].

We used the Monte Carlo simulation to determine the control coefficients of the proposed control chart because the theoretical derivation of the average run length is not tractable. The control coefficients k_1 and k_2 will be determined for various combinations of n and λ to yield the specified in-control average run length at $p=0.5$, say $ARL_0 = 370$. Then, the out-of-control ARLs will be calculated according to various values of p . The complete Monte Carlo simulation procedure is given as below.

1. (Setting up control limits)
 - 1.1. Select the sample size (or subgroup size) n and the smoothing constant λ .
 - 1.2. Select the initial values of k_1 and k_2 .
 - 1.3. Generate a random variable M_i for i -th subgroup from the binomial distribution having parameters n and $p=0.5$.
 - 1.4. Calculate $EWMA_{M_i}$ for i -th subgroup.
 - 1.5. Follow the procedure of the proposed control chart and check if the process is declared as out-of-control. If the process is declared as in-control, go to Step 1.3. If the process is declared as out-of-control, record the number of subgroups so far as the in-control run length.
 - 1.6. Repeat Steps 1.3 through 1.5 a sufficient number (10,000 say) of times to calculate the in-control ARL. If the in-control ARL is equal to the specified ARL_0 , then go to Step 2 with the current values of k_1 and k_2 . Otherwise, modify the values of k_1 and k_2 and repeat Steps 1.3 to 1.6.
2. (Evaluating the out-of-control ARL)
 - 2.1. Generate a random variable M_i for i -th subgroup from the binomial distribution with parameters n and $p=p_1 \neq 0.5$ considering a shift.
 - 2.2. Calculate $EWMA_{M_i}$ for i -th subgroup.
 - 2.3. Repeat 2.1 and 2.2 until the process is declared as out-of-control. Record the number of subgroups as an out-of-control run length.
 - 2.4. Repeat the above steps a sufficient number (10,000 say) of times to obtain the out-of-control ARL.

The values of the control coefficients for which ARL_0 is around 370 according to various values of sample size n and λ are placed in Table 1.

From Table 1, we note that there is no specific trend in control chart coefficients but we observed that the values of k_1 lie between 2 to 3 and the values of k_2 are below 1.

The values of ARL_1 for $n=9, 10, \dots, 25$ when $\lambda=0.05$ are obtained using the Monte Carlo simulation and placed in Table 2.

It is observed from Table 2 that the out-of-control ARL decreases rapidly as the value of p is away from 0.5. We

Download English Version:

<https://daneshyari.com/en/article/689036>

Download Persian Version:

<https://daneshyari.com/article/689036>

[Daneshyari.com](https://daneshyari.com)