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Chance-constrained model predictive control for drinking water networks

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ABSTRACT

This paper addresses a chance-constrained model predictive control (CC-MPC) strategy for the management of drinking water networks (DWNs) based on a finite horizon stochastic optimisation problem with joint probabilistic (chance) constraints. In this approach, water demands are considered additive stochastic disturbances with non-stationary uncertainty description, unbounded support and known (or approximated) quasi-concave probabilistic distribution. A deterministic equivalent of the stochastic problem is formulated using Boole's inequality to decompose joint chance constraints into single chance constraints and by considering a uniform allocation of risk to bound these later constraints. The resultant deterministic-equivalent optimisation problem is suitable to be solved with tractable quadratic programming (QP) or second order cone programming (SOCP) algorithms. The reformulation allows to explicitly and easily propagate uncertainty over the prediction horizon, and leads to a cost-efficient management of risk that consists in a dynamic back-off to avoid frequent violation of constraints. Results of applying the proposed approach to a real case study – the Barcelona DWN (Spain) – have shown that the network performance (in terms of operational costs) and the necessary back-off (to cope with stochastic disturbances) are optimised simultaneously within a single problem, keeping tractability of the solution, even in large-scale networks. The general formulation of the approach and the automatic computation of proper back-off within the MPC framework replace the need of experience-based heuristics or bi-level optimisation schemes that might compromise the trade-off between profits, reliability and computational burden.

1. Introduction

Drinking Water Networks (DWNs) form the link between urban water supply systems and drinking water consumers. These networks are vital for the normal functioning of modern society and maintaining a truly sustainable service is a must in these systems. All water supply undertakings should share a common purpose, stated in [1] as the achievement of the highest level of consumer satisfaction and service quality in line with the prevailing regulatory framework, whilst making best use of available resources. DWNs are large-scale multi-source/multi-sink flow systems that must be reliable and resilient while being subject to constraints and to continuously varying conditions with both deterministic and probabilistic nature. Customers behaviour determines the transport and storage operations within the network. Water use can vary in both the long and the short term, usually presenting time-based patterns for different areas. Therefore, a better understanding and forecasting of demands will improve both modelling and control of DWNs.

The growing complexity of these network systems, i.e., dimensionality, information structure constraints, non-linearities, uncertainty, and the higher performance requirements make these kind of problems costly to solve for real-time control applications and their optimal management is a task that has become an increasingly environmental and socio-economic research subject worldwide. Different approaches reported in the literature highlight the importance and development of the topic. As discussed in [2,3], during the last years, optimal operation of water supply systems has been addressed by a wide variety of methods. For example, in [4] a dynamic programming approach is proposed to generate pump schedules in real-time operation of a water supply system. The problem is solved by considering deterministic disturbances and decomposing the system in space and time to apply progressive optimality. In [2] a detailed review of several stochastic dynamic programming techniques applied to water reservoir operations is discussed, highlighting the curse of dimensionality of such techniques and proposing alternative methods to design cyclostationary daily control policies based on reinforcement

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learning. In [5] a critiquing-expert method that evaluates operating plans and provides feedback to the decision manager is proposed, which includes suggestions for improvement, warnings, and alternatives. In [6] a deterministic linear goal programming method is examined to aid in the identification of optimum operating policies for a multiple-reservoir system, highlighting the importance of the forecast quality to minimise pumping cost, but without robustness consideration. In [7] a combinatorial optimisation is proposed for the scheduling of on-off pumps assuming also deterministic and reliable forecast of demands. In [8] a centralised MPC strategy using mixed integer non-linear programming is proposed to regulate water volume in storage tanks and chlorine concentration, both around fixed targets, considering an anticipated consumer demand profile from historical data. Similarly, in [9–11] the economic operation of water networks is performed by using centralised MPC. They also assumed predicted disturbances as certain in the model but included a soft constraint to penalise depletions of water volume below a heuristic safety threshold without forcing any target regulation. An enhancement for these latter methods is proposed in [12], where MPC is combined in a hierarchical way with soft-computing methods and supply chain theory to compute dynamic safety stocks that cope with forecast uncertainty and achieve a self-tuning trade-off between economic optimisation and service reliability of a DWN.

Among the aforementioned approaches, decision policies based on the MPC framework [13], are suitable to face the operation of DWNs given their flexibility to manage constraints and to optimise multi-objective problems as the ones encountered in these complex systems, see [14]. The basic idea of MPC is to exploit a model of the system to predict its future evolution and compute control actions by optimising a desired cost function that takes into account such predictions; if future disturbances can be identified and described, a robust MPC can be developed to explicitly consider their effect on the future evolution of the controlled system.

Decision-making under uncertainty is a central issue in almost all disciplines and application areas. Since uncertainty in DWNs might be large and caused by many sources (e.g., exogenous and endogenous demands, noise, equipment degradation, plant model mismatch, other disturbances), it cannot be neglected in optimisation tasks if it is desired to fulfil reliability requirements and quality standards. In industrial practice, uncertainties are usually compensated by over-design of elements or overestimation of operational parameters by introducing safety factors obtained mostly by experience or application-dependent heuristics.

Most of the aforementioned operational MPC strategies for DWNs address uncertainty by solving a deterministic optimisation problem where, following the *certainty equivalence principle*, stochastic disturbances are replaced by their estimates based upon the information available at each time instant and assuming predictions as certain. This principle guarantees to obtain optimal control actions when using the expectation of disturbances, especially for linear models with small additive uncertainties, Gaussian distributions and quadratic cost functions. The MPC approaches following the aforesaid principle are often denoted *certainty equivalent* MPC (CE-MPC). Nevertheless, uncertainty in DWNs could be large, which avoids to take for granted that a certainty equivalence is justified for a reliable operation of the constrained system. Hence, the strategy is usually complemented with a (de)tuning of the controller, even though, CE-MPC can lead to poor performance and constraint violations due to the ignored effects of future uncertainty.

There is another widely reported class of control techniques that face uncertainties explicitly in the control law, named the *robust model predictive control* (RMPC). These strategies use an uncertain process model instead of a nominal one. A crucial factor in the design of these RMPC controllers is the characterisation of the uncertainty, which can be divided in two main paradigms: the deterministic worst-case description and the stochastic description. The approach relying on a purely deterministic unknown-but-bounded description of the uncertainty has prevailed in the robust control literature, see [15–19], but the main disadvantage of the related strategies is the conservatism of the resultant control policy that negatively affects the utility function of the DWNs operation. Moreover, in real applications the boundedness assumption of disturbances might not hold, hence, constraint violations are unavoidable due to unexpected events, faults, etc.

A more realistic description of uncertainty is the stochastic paradigm, which leads to less conservative control approaches by including explicit models of disturbances in the design of control laws and by transforming hard constraints into probabilistic constraints to cope with inevitable uncertainties. The stochastic approach is a classic one in the field of optimisation (see [20] for a review), but due to the advances in technology, which have improved computation capacity, and due to the flexibility of the MPC framework to incorporate models and constraints within an optimal control problem, a renewed attention has been given to the stochastic programming as a powerful tool for robust control design, leading to the *Stochastic MPC*, which has a particular variant called *Chance-Constrained MPC* (CC-MPC) [21]. This stochastic control strategy describes robustness in terms of probabilistic (chance) constraints [22], which require that the violation probability of any operational requirement or physical constraint is below a prescribed value, representing the notion of reliability or risk of the DWN. By setting this value properly, the operator can trade conservatism against performance. Relevant works that address the CC-MPC approach in water systems can be found in [23,24] and references therein. Other stochastic approaches currently researched are the ones based on multi-stage stochastic programming (MSP) methods, e.g., scenario-based stochastic programming, sampling stochastic dynamic programming, interval stochastic programming, among others [25,26]. The main limitation of these latter approaches is their narrowed applicability to large-scale stochastic models, which might be a cumbersome task, especially when several disturbances, sources of uncertainty and decision vectors of large dimension are involved. In fact, most of the case studies reported for real-time optimisation of water systems and other applications are small-scale problems, hence, the vast portfolio of developed stochastic techniques has been dedicated for long-term off-line planning of operations or for networks design.

The main contribution of this paper is the introduction of a formal RMPC formulation for the management of DWNs based on chance constraint programming. The paper presents the results of applying an economic CC-MPC to optimise the operation of flow networks, especially those related to the transport and storage of potable water, seeking to achieve a specified customer service level and a reliable DWN. The complexity of the stochastic problem is addressed by using an analytical approximation of the chance constraints to reformulate the problem into a tractable deterministic equivalent by using Boole's inequality, a uniform risk allocation policy and the stochastic characteristics of disturbance forecasts. This systematic approach keeps the convex nature of the multi-objective constrained finite horizon optimisation problem and brings other practical benefits, i.e., flexibility, reliability, and tractability of the reformulated CC-MPC controller as a decision-support tool, which are shown in this paper through a real case study: the Barcelona DWN. The approach avoids the set-up of conservative and heuristic thresholds or bi-level optimisation approaches [27] for safety volumes in water storage tanks. In fact, the robustness of the CC-MPC leads to a cost-efficient management of a dynamic back-off (uncertainty-aware time-varying safety volumes) to avoid frequent violation of constraints.