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Profitable and dynamically feasible operating point selection for constrained processes

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ABSTRACT

The operating point of a typical chemical process is determined by solving a non-linear optimization problem where the objective is to minimize an economic cost subject to constraints. Often, some or all of the constraints at the optimal solution are active, i.e., the solution is constrained. Though it is profitable to operate at the constrained optimal point, it might lead to infeasible operation due to uncertainties. Hence, industries try to operate the plant close to the optimal point by "backing-off" to achieve the desired economic benefits. Therefore, the primary focus of this paper is to present an optimization formulation for solving the dynamic back-off problem based on an economic cost function. In this regard, we work within a stochastic framework that ensures feasible dynamic operating region within the prescribed confidence limit. In this work, we aim to reduce the economic loss due to the back-off by simultaneously solving for the operating point and a compatible controller that ensures feasibility. Since the resulting formulation is non-linear and non-convex, we propose a novel two-stage iterative solution procedure such that a convex problem is solved at each step in the iteration. Finally, the proposed approach is demonstrated using case studies.

1. Background

Profitability is the major concern of a chemical plant and one approach to achieve this is to operate the plant at the optimal point obtained from a non-linear steady state optimizer. The optimizer minimizes a suitable cost function subject to equality and inequality constraints. Often, the solution of the optimizer is constrained at some of the inequalities, that is, there are several active constraints. Typically, it is assumed that these active constraints should be controlled at their limiting values to achieve economic benefits. However, the presence of uncertainties in the form of measurement noise, modeling error, parametric uncertainties and disturbances might cause constraint violations. Therefore, it is important to find an operating point close to the active constraints such that the plant remains feasible for the expected range of uncertainties. Thus, the focus of our work is to propose an optimization formulation that obtains the best trading-off between feasibility and profitability.

Optimal process operations depend on process design and safety thresholds, etc. These constraints define the feasible operating window to the optimizer. To ensure feasible operation under uncertain conditions, it may be necessary to "back-off" from the active constraints which however results in loss of achievable profit. Hence, the optimizer minimizes a loss function for backing – off from the active constraints. The term "back – off" is defined as,

$$\begin{aligned} \mathit{Back-off} = {} & |\mathit{Actual\ steady\ state\ operating\ point} \\ & - \mathit{Nominally\ optimal\ steady\ state\ operating\ point}| \end{aligned} \quad (1)$$

Based on the notion of back-off, Narraway et al. [15] presented a method to assess the economic performance of the plant in the presence of disturbances. To ensure feasibility, the maximum amplitude of the disturbance for a certain range of frequency was used to determine the necessary back-off and alternate designs were evaluated. They assume the set of measurements are perfectly controlled and controllability is tested after obtaining the solution. Later, Narraway and Perkins [16] extended their frequency response based method of estimating the closed loop constraint back off on the assumption of perfect control hypothesis to select the optimal set of measurements and manipulated inputs. This was accomplished by introducing the binary decision variable into the bounds of all possible measurements and manipulations. Also, the method was extended for the case of realistic PI controllers. Although the formulation is an Mixed Integer Linear Program

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(MILP), the dimension of the problem is very high owing to the number of frequencies considered for each of the constraints. To solve this, a solution algorithm was presented where the obtained solution is compared with the open loop (without control) solution to quantify the profitability that would achieved by the controller and the controller with less benefits are eliminated [7]. All of the above methods were developed to handle single disturbance only.

To address the case of multiple disturbances, Bahri et al. [2] addressed the back off problem for control of active constraints in the regulatory layer by solving the open loop problem. Figueroa et al. [5] extended the above approach to the closed-loop case where the figure of merit "maximum percentage recovery" is defined to choose between alternative control configurations. In summary, disturbance is the only source of uncertainty considered in evaluating the different control structures. However, in some cases measurement noise and control error also play a significant role.

Disturbances are typically categorized based on the time scale or frequency of occurrence as fast or high-frequency disturbance and slow or low-frequency disturbance. The lower regulatory layer generally handles the fast disturbances whereas the slow disturbances are handled by the steady state optimizer. The objective of the optimization layer is to provide set points to the control layer. These set points depend on the set of design variables and measurements selected for estimating the model parameters. And, the choice of measurements have a profound impact in the steady state economics. In this regard, de Hennin et al. [8] presented a method for estimating the likely economic benefit that could be achieved by implementing a steady state optimizer. The cost of instrumentation is also included in addition to the operational cost to determine the best optimal measurements. Some of the other works that use the concept of economic back-off in the area of controllability analysis are Young et al. [20] and Bahri et al. [1].

Loeblein and Perkins [10] proposed a measure of average deviation from optimum that allows the estimation of economic value of different online optimization structures. In addition to measurement selection, their work addressed the impact of model uncertainty on the economics of the optimizer. To analyze this issue, the authors considered a simple model, approximate model and rigorous model and concluded that approximate model is appropriate for on-line optimization. Later, Loeblein and Perkins [11,12] extended their method of average deviation from optimum to analyze the dynamic economics of regulatory layer which is assumed to be implemented using Model Predictive Control (MPC) system. However, fixed control structures are assumed to rank between the alternatives.

Peng et al. [19] proposed a stochastic formulation for the determination of back-off points based on the notion of expected dynamic operating region. The basic idea in their approach is that the simultaneous selection of controller and back off point will find a optimal controller that minimizes the variability of the active constrained variables. Since the disturbances are assumed to be stochastic, the dynamic operation is defined in terms of variance. Extensions of the method to discrete time and partial state information case do not alter the formulation. Despite this, the final form of the optimization problem contains a set of reverse convex constraints which make the problem difficult to solve. Therefore, a branch and bound type algorithm was proposed. Sensor selection for control purposes are addressed in this framework [14,18]. Chmielewski and Manthanwar [4] have found that the obtained optimal multivariable feedback controller can be used to tune the objective function weights of the MPC controller.

In this work, we propose a stochastic formulation of the dynamic back-off problem that ensures feasible operation for the prescribed confidence limit. Following Peng et al. [19], the dynamic operating region is defined for the given disturbances which follow from the closed loop covariance analysis of the state space model of the process. Under rather general conditions, this dynamic region can be characterized as an ellipsoid. The loss function, is a measure of departure from optimality and we develop a theoretically and conceptually sound loss function. Controller selection also plays a crucial role in shaping the dynamic operating region while the size of the region is characterized by the prescribed confidence limit and variance of the disturbance considered. Thus, consideration of the controller gain as a decision variable is important in determining the optimal operating point which minimizes the loss in profit. Therefore, the focus of our work is to propose an optimization formulation that determines the economic backed-off operating point by finding at the same time a suitable controller gain.

The current formulation contains an explicit representation of the ellipsoid to describe the system dynamics and can handle partially constrained cases. Unlike our previous work [13], in the current formulation, the back-off can be viewed as a slack variable in the feasibility constraints. Furthermore, a novel and computationally efficient solution methodology has been presented to solve the non-linear non-convex problem.

This paper is organized as follows. In the next section, we define the problem and present a development of stochastic formulation and convex relaxations of the constraints. Next, a solution algorithm has been developed. Finally, illustrations are provided to demonstrate the approach.

2. Formulation of dynamic back-off problem

The objective of this section is to present an optimization formulation that determines the most profitable steady state operating point given that the plant has to remain feasible for the expected set of disturbances affecting the process. Hence, the optimization formulation should also include differential constraints that characterize the dynamic operating region of the plant. The feasibility becomes an important issue while operating the plant at the constrained optimal point. Therefore, we need to solve a dynamic back-off problem.

2.1. Optimization formulation

We start by determining the Optimal steady state Operating Point (OOP) by minimizing the economic cost (the negative of the operating profit) $J(x_0, u_0, \bar{d}_0)$ where x_0, u_0 and $\bar{d}_0$ denote the states, manipulated inputs and nominal value of disturbances. Thus, the steady state optimizer solves the non-linear steady state optimization problem of the form,

$$\min_{x_0, u_0} \quad J(x_0, u_0, \bar{d}_0) \tag{2a}$$

$$s.t. \quad g(x_0, u_0, \bar{d}_0) = 0 \tag{2b}$$

$$h(x_0, u_0, \bar{d}_0) \leq 0 \tag{2c}$$

At OOP, the states and manipulated inputs are denoted as x_0^* and u_0^*, respectively. At OOP, there are three possible cases: unconstrained optimum (no active constraints), partially constrained (the number of active constraints is less than the number of manipulated inputs) and fully constrained (the number of active constraints equals the number of manipulated inputs). Peng et al. [19] have addressed the problem for fully constrained case and the back-off from the linearized optimal solution is determined. In the present work, the focus is on the more general partially constrained case. In contrast to the fully constrained case where a linear approximation of the cost function around the optimal point is valid, the partially constrained case requires one to include a quadratic penalty for the inputs to account for the unconstrained degrees of freedom.