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Proportional stabilization and closed-loop identification of an unstable fractional order process

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ABSTRACT

This paper deals with proportional stabilization and closed-loop step response identification of the fractional order counterparts of the unstable first order plus dead time (FOPDT) processes. At first, the necessary and sufficient condition for stabilizability of such processes by proportional controllers is found. Then, by assuming that a process of this kind has been stabilized by a proportional controller and the step response data of the closed-loop system is available, an algorithm is proposed for estimating the order and the parameters of an unstable fractional order model by using the mentioned data.

1. Introduction

The inherent nonlinearity of industrial processes yields in these systems can have multiple steady states [1,2]. Some of these steady states, which may be desired operating points for the process, can be unstable [1]. Some typical processes with this property are bioreactors, polymerization reactors, exothermic reactors, batch reactors, steam boilers, distillation columns, crystallization processes, and biological systems [3–6].To describe the behavior of these processes around their unstable steady states, unstable linear time invariant models are generally used [1]. Using fractional order operators, which are originated from the fractional calculus, in constructing the models is a way to enrich these models for better describing the behavior of real-world processes. For example, it has been verified that fractional order models can be effectively used in modeling of isotope separation columns [7], bioreactors [8], pressurized heavy water reactors [9], liquid/liquid interfaces [10], biological systems [11], thermal systems [12,13], and hydrologic processes [14]. One significant motivation for applying simple fractional order models in such applications is that these models can well approximate the dynamics of many high-order classical models [15]. Therefore, simple fractional order models can be good candidates for describing the dynamics of those systems which have been conventionally modeled by high-order classical models. Due to this advantage of fractional order models, finding appropriate methods for estimating the order and the parameters of such models is of great importance. Among different methods, those methods which are based on using simple-achieved data of the system would be valuable methods. For instance, those methods in which the step response data of the system is used can be appropriate in practical point of view [16]. Such data can be achieved by doing simple experiments. In [17], some methods have been proposed for estimating the order and the parameters of stable fractional order models by using the step response data. To complete the mentioned work, the aim of this paper is to propose an appropriate method for estimating the order and the parameters of an unstable fractional order model approximating the dynamics of an unstable process. To achieve this aim, at first the unstable process is stabilized by a proportional controller, and then the step response data of the closed-loop system is used by the proposed method to estimate the order and the parameters of an unstable fractional order model. Since for getting the required estimation data it is necessary to stabilize the process, the stabilizability problem is also investigated in the present work. The considered model in this paper is

$$G(s) = \frac{ke^{-Ls}}{Ts^{\alpha} - 1} \quad \text{where } 0 < \alpha < 1,\ T > 0,\ \text{and } k \neq 0, \tag{1}$$

which can be considered as the fractional counterpart of the unstable first-order plus dead time (FOPDT) models [18]. More precisely, the following questions will be answered in this paper:

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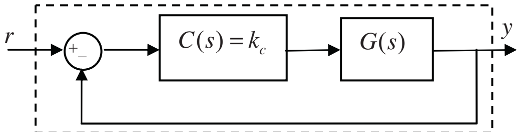

Fig. 1. A simple feedback control system structure.

(1) What is the necessary and sufficient condition for stabilizability of a system modeled in the form (1) by a proportional controller?
(2) If a system in the form (1) is stabilized by a proportional controller, how the order and the parameters of this system can be estimated from the noisy step response data of the closed-loop system?

The first question will be answered in Section 2. Section 3 devotes to finding the answer of the second question. Finally, the paper is concluded in Section 4.

2. Stabilizability by a proportional controller

Consider the control system structure shown in Fig. 1, and assume that the process in this structure can be modeled by the unstable fractional order transfer function $G(s)$ in form (1). In this section, it is investigated that is there a proportional controller in the form $C(s) = k_c$ which guarantees the stability of the closed-loop system shown in Fig. 1? To this end, at first consider the following theorem.

Theorem 1. System

$$G(s) = \frac{e^{-Ls}}{s^{\alpha} - 1}, \tag{2}$$

where $0 < \alpha < 1$ is stabilizable by a proportional controller if and only if

$$L < \pi(1-\alpha)\left(2\cos\left(\frac{\alpha\pi}{2}\right)\right)^{-1/\alpha}. \tag{3}$$

Proof. Consider the closed-loop system shown in Fig. 1 where $G(s)$ is given by (2). According to [19: Theorem 3.1], we know that this closed-loop system is BIBO stable if and only if it does not have any pole in region $\{s \in \mathbb{C} | Re(s) \geq 0\}$. In the other words, this closed-loop system is BIBO stable if equation

$$1 + k_c G(s) = 0, \tag{4}$$

where $G(s)$ is given by (2), does not have any solution in region $\{s \in \mathbb{C} | Re(s) \geq 0\}$. Therefore, the stability of the mentioned closed-loop system can be checked by the Nyquist stability criterion [20]. To this end, let us investigate the shape of the polar plot of $G(j\omega)$. From (2) it is concluded that

$$G(j\omega) = \frac{\cos(L\omega) - j\sin(L\omega)}{\omega^{\alpha}\cos(\alpha\pi/2) + j\omega^{\alpha}\sin(\alpha\pi/2) - 1}, \tag{5}$$

for $\omega \geq 0$. (5) results in

$$\varphi(\omega) \triangleq \measuredangle G(j\omega) = \begin{cases} -L\omega + \tan^{-1}\dfrac{\omega^{\alpha}\sin(\alpha\pi/2)}{1-\omega^{\alpha}\cos(\alpha\pi/2)} - \pi & \text{if} \quad \omega^{\alpha}\cos(\alpha\pi/2) < 1 \\ -L\omega - \dfrac{\pi}{2} & \text{if} \quad \omega^{\alpha}\cos(\alpha\pi/2) = 1 \\ -L\omega + \tan^{-1}\dfrac{\omega^{\alpha}\sin(\alpha\pi/2)}{1-\omega^{\alpha}\cos(\alpha\pi/2)} & \text{if} \quad \omega^{\alpha}\cos(\alpha\pi/2) > 1 \end{cases}, \tag{6}$$

According to (6),

$$\frac{d\varphi(\omega)}{d\omega} = -L + \frac{\alpha\omega^{\alpha-1}\sin(\alpha\pi/2)}{1 - 2\omega^{\alpha}\cos(\alpha\pi/2) + \omega^{2\alpha}}. \tag{7}$$

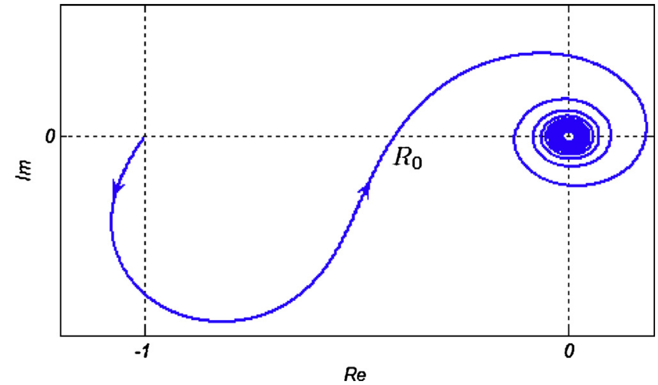

Fig. 2. Polar plot of $G(j\omega)$ where $L < L^*$.

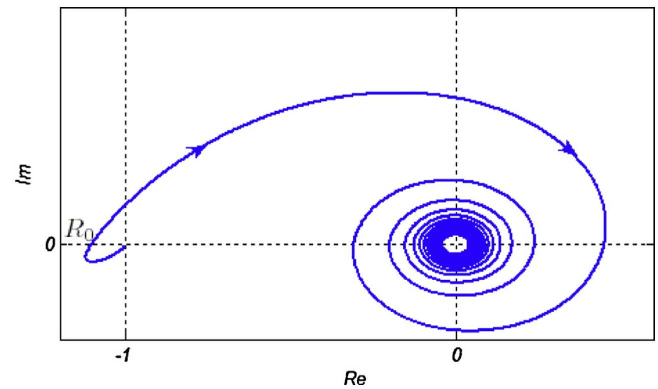

Fig. 3. Polar plot of $G(j\omega)$ where $L > L^*$.

Since $0 < \alpha < 1$, it is deduced that $\lim_{\omega \to 0^+}(d\varphi(\omega)/d\omega) = +\infty$. This means that $\varphi(\omega)$ will be an increasing function for small values of $\omega \geq 0$. On the other hand, according to (7) we have $\lim_{\omega \to +\infty}(d\varphi/d\omega) = -\infty$. Hence, there is $\omega = \omega_0$ such that $\varphi(\omega)$ is a decreasing function for $\omega > \omega_0$. Paying attention to the mentioned points, two different cases schematically shown in Figs. 2 and 3 may occur for the polar plot of $G(j\omega)$. In the first case (Fig. 2), the polar plot begins from point $(-1,0)$, and after that intersects the real axis at a point denoted by $(R_0,0)$ which places in the right side of the beginning point $(-1,0)$. According to the Nyquist stability criterion, in such a case system (2) is stabilized by proportional controller $C(s) = k_c$ if $1 < k_c < 1/R_0$. In the second case (Fig. 3), the polar plot begins from point $(-1,0)$ and after that intersects the real axis at a point denoted by $(R_0,0)$ which settles in the left side of the beginning point $(-1,0)$. According to the Nyquist stability criterion, it is resulted that in this case system (2) cannot be stabilized by a proportional controller. Now, we want to find the critical time delay by which the polar plot of $G(j\omega)$ varies from the first case to the second case. For this critical time delay,