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Information theoretic multiscale truncated SVD for multilead electrocardiogram

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ABSTRACT

Background and objective: In this paper an information theory based multiscale singular value decomposition (SVD) is proposed for multilead electrocardiogram (ECG) signal processing. The shrinkage of singular values for different multivariate multiscale matrices at wavelet scales is based on information content. It aims to capture and preserve the information of clinically important local waves like P-waves, Q-waves, T-waves and QRS-complexes.

Methods: The information is derived through clinically relevant multivariate multiscale entropy in SVD domain modifying Shannon's entropy. This optimizes the approximate ranks for matrices to capture the clinical components of ECG signals appearing at different scales. A newly introduced multivariate clinical distortion (MCD) metric is computed and compared with existing subjective and objective signal distortion measures. The proposed method is tested with records from CSE multilead measurement library and PTB diagnostic ECG database for various pathological cases.

Results: It gives average percentage root mean square difference (PRD), average normalized root mean square error (NRMSE), average wavelet energy based diagnostic distortion measure (WEDD) values 5.8879%, 0.0059 and 1.0760% respectively for myocarditis pathology. The corresponding MCD value is 1.9429%. The highest average PRD and average WEDD values are 11.4053% and 5.5194% for cardiomyopathy with the corresponding MCD value 1.4003%.

Conclusions: Based on WEDD values and mean opinion scores (MOS), the quality group of all processed signals fall under excellent category.

1. Introduction

Cardiologists normally look at electrocardiogram (ECG) for clinically essential diagnostic information like, P-wave, QRS-complex, T-wave, Q-wave, S-wave, ST-segments with innate morphologies, durations and amplitudes. This helps investigating cardiac pathology. For decades, numerous studies and ideas are applied for better extraction of information about these components. When recorded, these types of physiological signals are likely to have deterministic and stochastic components [1]. To meet the demand for acceptable 'PQRST' morphology in a clinical setting, researchers extract relevant clinical information using different methods. Among them, the singular value decomposition (SVD) is applied to the fetal ECG extraction [2], noise removal and recovery of lost

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channel [3], data compression [4], denoising sensors arrays data [5] and denoising of multilead ECG signals [6]. It is employed for features extraction [7] for support vector machine (SVM) to classify the different physiological states. For pathological ECG beats recognition, SVD is used in [8]. The selection of singular values plays an important role for SVD based methods in capturing clinical information in processed signals. In our earlier study, to select singular values multiscale root fractional energy contribution is considered for multilead ECG data compression [9].

In another type of approach to extract the complexity of physiological signal, Pincus introduced approximate entropy (ApEn) [10]. The sample entropy (SampEn) was introduced by Richman and Moorman [11] to measure the regularity of complex physiological univariate data. The Higher SampEn metric indicates higher complexity. It is maximized for a randomized process and yields lower value for more regular data. It has been reported to yield misleading results [1,12,13]. The ECG signals of healthy subjects with heart rate variability (HRV) show lower SampEn compared to atrial fibrillation. The ApEn and the SampEn metrics are based on the single-scale. To extract information at multiple time scales, the SampEn is evaluated at multiple scale [1,14–17] by the coarse-graining procedure as multiscale entropy. This was modified to evaluate the multivariate sample entropy [12,13]. These types of definition of entropy of physiological signals mainly quantify the complexity. Instead, if classical Shannon entropy is assigned to quantify the information, it gives higher value for a completely random system and lower value for more ordered time series [1,14,15] which contains local waves.

In this work, the information presence in physiological signal is captured in multiscale SVD domain. As the singular values capture the information of the signal in SVD domain, it is suggested to evaluate clinically relevant modified Shannon's multivariate multiscale entropy to represent clinical information. It is expected that in time and space, the entropy may provide better information about the recorded physiological signals which contain local waves and random noise. So, in a multiscale SVD domain, modified Shannon's multivariate multiscale entropy (MME) for multichannel ECG data, is derived. Based on the MME, suitable shrinkage of singular values for multiscale SVD is formulated to capture the maximum clinical information. In an ECG signal P-waves, Q-waves, T-waves and QRS-complexes are desired clinical information. This shrinkage of singular value (SV) gives minimum clinical component distortion. In the rest of the article, in Section 2, the method is explained and in Section 3, results are summarized. At Section 4, a brief conclusion is drawn.

2. Method

To formulate entropy based multiscale truncated SVD using multiresolution analysis, multilead electrocardiogram signals: lead-I, lead-II, lead-III, lead-aVR, lead-aVL, lead-aVF, lead-V1, lead-V2, lead-V3, lead-V4, lead-V5 and lead-V6 are considered. Though among these only 8 leads are linearly independent, standard 12 leads are used to formulate multivariate analysis. These signals are individually wavelet transformed using the same mother wavelet and decomposition levels [18]. The details subbands are denoted as cD_j, where decomposition level $j=1, 2, \ldots, L$ and the approximation subband is cA_L. The wavelet decomposition level is L. To form multivariate matrices at different decomposition levels, similar subbands from different ECG leads are collected. At approximation level, wavelet coefficients of approximation subbands of all the leads are arranged in columns to form matrix $\mathbf{A}_L \in R^{N_L \times n}$. Similarly, at the details, wavelet coefficients of detail subbands of all the leads are arranged in columns to form matrices $\mathbf{D}_j \in R^{N_j \times n}$. The number of wavelet coefficients in each column at approximations is N_L and in each column at details are N_j respectively. The n is the number of columns which also represents number of ECG leads. The SVD of approximation matrix is given as

$$\mathrm{A_L} = \mathrm{U_{A_L}}\,\Sigma_{\mathrm{A_L}}\,\mathrm{V^T_{A_L}} \tag{1}$$

where $\mathrm{U_{A_L}}$, $\mathrm{V^T_{A_L}}$ and $\Sigma_{\mathrm{A_L}}$ are left singular, right singular and diagonal singular value matrices respectively. For matrices at details, the SVD's are given as

$$\mathrm{D_j} = \mathrm{U_{D_j}}\,\Sigma_{\mathrm{D_j}}\,\mathrm{V^T_{D_j}} \tag{2}$$

where $\mathrm{U_{D_j}}$, $\mathrm{V^T_{D_j}}$ and $\Sigma_{\mathrm{D_j}}$ are left singular, right singular and diagonal singular value matrices respectively. The rank of $\mathbf{A}_L$ is $rank(A_L) = n^{A_L}$ and ranks of $\mathbf{D}_j$ are $rank(D_j) = n^{D_j}$. These ranks represent the independent columns containing coefficients of subbands of different ECG leads at wavelet scales. The diagonal entries of singular value matrices for approximation and details levels are given as

$$\sigma_i^{A_L} \Rightarrow \sigma_1^{A_L} \geq \sigma_2^{A_L} \geq \cdots \geq \sigma_n^{A_L} \geq 0 \tag{3}$$

$$\sigma_i^{D_j} \Rightarrow \sigma_1^{D_j} \geq \sigma_2^{D_j} \geq \cdots \geq \sigma_n^{D_j} \geq 0 \tag{4}$$

where $i=1, 2, \ldots, n$. It is expected to get low rank estimations for the matrices at approximation and detail levels, $rank(\widehat{\mathrm{A_L}}) = k^{A_L}$ and $rank(\widehat{\mathrm{D_j}}) = k^{D_j}$ from singular value matrices to retain maximum signal information, where $k^{A_L} < n^{A_L}$ and $k^{D_j} < n^{D_j}$. Thus, the truncated SVD solution of approximation matrix and details matrices are given as

$$\widehat{\mathrm{A_L}} = \mathrm{U_{k^{A_L}}}\,\Sigma_{\mathrm{k^{A_L}}}\,\mathrm{V^T_{k^{A_L}}} \tag{5}$$

$$\widehat{\mathrm{D_j}} = \mathrm{U_{k^{D_j}}}\,\Sigma_{\mathrm{k^{D_j}}}\,\mathrm{V^T_{k^{D_j}}} \tag{6}$$

where k^{A_L} and k^{D_j} singular values those are selected from the columns of left and right unitary matrices. In reconstructing signal, if too small rank is selected, all subtlety is lost. The errors $||\mathrm{A_L} - \widehat{\mathrm{A_L}}||_F$ and $||\mathrm{D_j} - \widehat{\mathrm{D_j}}||_F$ may be minimized if singular values are suitably chosen. $||\,.\,||_F$ represents Frobenius norm. It is proposed that the information theoretic based shrinkage of singular values may provide a suitable solution by minimizing errors and optimizing clinical information content in output signals. The SVD evaluated in time and space is expected to preserve information of local waves. This may be achieved if the selection of suitable number of singular values are derived weighting with information and energy for different matrices, where $k^{A_L} < n^{A_L}$ and $k^{D_j} < n^{D_j}$. After reducing dimensions through multiscale SVD operation, individual ECG leads are