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An extremum seeking approach via variable-structure control for fed-batch bioreactors with uncertain growth rate

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ABSTRACT

In this paper, we present an extremum-seeking scheme based on an approach to variable structure control for fed-batch bioreactors. The proposed scheme deals with uncertainty on the specific growth rate without assuming an explicit mathematical expression. The control approach exploits the inhibitory effect of the substrate concentration on the growth rate, in such a manner that the closed-loop system reaches the sliding regime on an optimal switching manifold, which is defined by maximizing biomass production. The control scheme comprises an estimation scheme consisting of a high-gain observer and a discrete gradient estimator which computes the unknown terms. The practical stabilizability for the closed-loop system around an unknown optimal set-point is analyzed. Numerical experiments illustrate the effectiveness of the proposed approach.

1. Introduction

The fed-batch bioreactor strategy is typically used to reach a high cell density within the vessel for different purposes such as to produce some chemical compounds, synthesized by microorganism in fermentative processes, or to cultivate biomass for the utilization or extraction of its metabolites [4,5,24]. The optimal production of the synthesized product (e.g., pharmaceutical products, enzymes, proteins, etc.) or biomass (for example, baker's yeast, *Lactobacillus casei*, among others) is one of the key issues in the operation of fed-batch bioreactors [4]. However, the optimization of the operating conditions of bioreactors can be complicated to be reached, mainly due to highly nonlinear nature and the substantial unmodeled dynamics [26]. Also, the optimal operating conditions in bioreactors can be riskier; i.e., the effect of the external perturbations or small changes in the bioreactor environment can lead towards undesirable operating conditions [14,15,18,26]. In fact, it has been recently demonstrated that the substrate inhibitory effect provokes structural instability in the dynamical behavior of the bioprocess [18]. Hence, the fed-batch bioreactors are suitable candidates for using optimal and robust stabilization schemes.

Usually, the control schemes for bioreactors are focused on the regulation, servo set-points, or tracking reference trajectories [3,15,19,22,30]. However, in many biotechnological applications the control objective is to optimize an objective function that can be a function of unknown parameters in order to keep a performance variable at its optimal value [5,6]. Also, it is common that the explicit form of the performance function be unavailable or highly uncertain (e.g., the growth rate), and the performance function can be subject to bounded time-varying disturbances due to the effect of variation in the environmental variables (for instance, dissolved oxygen, temperature, or pH) [5,30]. In this way, perturbation-based (nonmodel-based) and model-based extremum seeking controls are two methods to handle these kinds of optimization problems [5,6]. The goal of extremum seeking schemes is to find the operating set-points, a priori unknown, such that an objective function (possibly subject to uncertainties) reaches their extremum value [13,28].

Complementarily, form the optimal viewpoint the extremum seeking control schemes have been an active research area with distinct application issues. The applications of these methods include, for example: the adjustment of radio telescope antennas in order to maximize the received signal; blade adjustment in water turbines or wind mills to maximize the generated power, and in anti-lock braking system (ABS) control to lead the maximal value of the tire/road friction force to be reached during emergency braking [7,9,10,12,21,29]. Also, an intensive research

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activity has been devoted to adaptive extremum seeking control schemes applied to bioprocesses (see e.g., [4–6,13,24,25,28,30,31]). In [4,5,24,30], adaptive extremum seeking approaches have been developed in order to drive the bioprocess states to the desired unknown set-point where the biomass production is maximized. The adaptive extremum schemes are based on parameter learning laws for unknown parameters estimation, and a dither signal to ensure the convergence to a neighborhood of its optimal value [5,30]. However, the model-based adaptive extremum seeking algorithms require prior information about: (a) the model for the biomass growth rate (as Haldane, Monod, or Cointois model) and (b) bounds of the parameters which, in most biochemical processes, is hard to obtain from available data [5,6,30]. Also, nonmodel-based adaptive extremum-seeking control methods have been applicable to diverse control systems with local minimum (or maximum) that defines their optimal operating condition (see the seminal work of Krstic [17] and [30]). These approaches employ one dynamic feedback composed by a sinusoidal perturbation signal, and adaptive extremum searching to find an unknown optimal operating condition for the plant. On the other hand, the extremum-seeking control problem has also been studied in the sliding mode control framework [7,9,12,16,29]. In such contributions, the main idea is to ensure that a desired output follows an increasing time function as close as possible to its extremal value via discontinuous controls and sliding mode motions [11,21]. Nevertheless, as far as we know, there is no extreme seeking schemes based on sliding mode theory for bioprocesses.

In this paper, we propose an extremum-seeking control scheme based on variable-structure theory such that optimal stabilization for a class of fed-batch bioreactors will be achieved. The first step in the controller design is an "ideal" optimum seeking controller which is found from the variable-structure theory [23]. The ideal control allows us to reach a sliding regime on an optimal manifold defined in the sense of maximizing the biomass production such that the states of the closed-loop system are driven to the optimum operation condition within the domain. In the second step, a high-gain observer-based uncertain estimator is used to approach the unknown terms in the ideal control law. In this stage of design, we show that the exact knowledge of the derivative of the reaction product yielding rate with respect to the substrate is needed to compute the optimal manifold. After that, as a final design step, we show that conditions in second step can be solved using the discrete gradient estimator proposed in [9,10]. Thus, the practical stabilization in the neighborhood of the unknown optimal set-point is ensured when the estimator scheme and controller are coupled. That is, the proposed scheme comprises the variable structure feedback and an uncertain dynamic estimator. Unlike the previous sliding-mode based extremum seeking schemes [7,9,10,12,16,29], the proposed control approach is based upon the equivalent control method [23], and the feedback structure is designed to achieve the practical stabilization at some optimal manifold (switching manifold), without using an increasing time function as the output reference. In order to evaluate the effectiveness of the proposed approach, numerical experiments were carried out considering the uncertainty on the growth rate and load disturbance from the inlet substrate concentration.

The rest of the paper is organized as follows: In Section 2 the dynamic model for the fed-batch bioreactor is presented and the control problem is formulated. Section 3 contains the design of the extremum seeking controller and the stability analysis of the closed-loop system. Numerical experiments that illustrate the performance and robustness of the proposed control approach are shown in Section 4. Finally, some concluding remarks are discussed in Section 5.

2. Model description and problem formulation

Consider a biomass culture processes occurring within a fed-batch bioreactor, where the microorganisms X grow by consuming a substrate S. Such a bioreaction is a first approximation of a more complicated biochemical reaction network and can be written as:

$$S \overset{\mu(\cdot)x}{\hookleftarrow} X$$

The growth rate is denoted by $\mu(\cdot)x$, where $\mu : \mathbb{R}_+ \to \mathbb{R}$, is a smooth function and the symbol $\hookleftarrow$ indicates that the biomass X is an autocatalyst; i.e., the microorganisms are at the same time products and catalysts. The following dynamical model can be found from a mass balance into the bioreactor:

$$\dot{s} = u(s_f - s) - k_1\mu(\cdot)x \tag{1}$$

$$\dot{x} = \mu(\cdot)x - ux \tag{2}$$

$$\dot{v} = uv \tag{3}$$

The state vector $z = [s, x, v]^T \in \mathbb{R}^3$ has components representing the concentrations of substrate, biomass and volume of the liquid medium in the vessel, respectively (the superscript T stands for transpose); the control input u is called dilution rate, i.e., the ratio between the feeding flow with respect to the reaction volume; s_f denotes the inlet substrate concentration; and $k_1 > 0$ is a yield coefficient. Now, with respect to the stability properties for open-loop bioreactor model we depart from the following result.

Theorem 1. [3] *Let $z = [s, x, v]^T \in \mathbb{R}^3$ be the state vector of the system (1)–(3), if the dilution rate is a bounded signal, i.e., $0 \le u_{min} \le u(t) \forall t \ge 0$, then the state variables are positive and bounded for all t.*

Corollary 1. *Let us consider the set $\Omega = \{(s, x, v) \in \mathbb{R}^3 | 0 \le s \le s_{\max}, 0 \le x \le x_{\max}, \underline{v} \le v \le \overline{v}\}$ with $s_{\max}, x_{\max}, \overline{v} < \infty$, then the set Ω is positively invariant with respect to the vector field (1)–(3).*

Proof. Straightforward from the Bounded Input Bounded State stability property of (1)–(3) (see Chapter 1 of [3]). □

In order to provide more generality in the controller proposed, we imposed the following properties on the specific growth rate μ:

Property 1. *$\mu \in C^\infty(S)$, where $S = \{s \in \mathbb{R} | 0 \le s \le s_m\}$ with $s_m < \infty$; and there exist a value $s^* \in S$ such that $\mu \le \mu(s*) \triangleq \overline{\mu} \in \mathbb{R} \forall s \in S$, with $\overline{\mu} < \infty$ as the upper bound of μ.*

Property 2 (Concavity property). *The first derivative of μ with respect to s, denoted by μ' satisfies the follows: (a) $\mu' > 0 \forall \tilde{s} < s*$; $\mu' = 0$ at $\tilde{s} = s*$ and; (b) $\mu' < 0 \forall \tilde{s} > s*$, where $\tilde{s} \in S$.*

Remark 1. Properties 1 and 2 hold independently of the kinetics governing the bioreaction. Property 1 regards the existence of the parameter named maximum growth rate $\overline{\mu} < \infty$, which has a very important biochemical meaning. Property 2 (Concavity property) is more closely related to mathematical concepts about nonmonotonic continuous functions with a local maximum (e.g., Haldane kinetics [26]).

A major difficulty in the monitoring and control of bioprocesses is the lack of reliable and simple sensors for following the evolution of the key state variables and parameters such as biomass and growth rate. In fact, a typical situation in bioreactor applications is when the biomass concentration is not available for on-line measurement while the product gaseous outflow rate (e.g., CO_2 in fermentation processes), is easier to measure online [5]. Now, it is well known that the product outflow rate can be modeled as [3]:

$$Q = k_2\mu(\cdot)x \tag{4}$$