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Decentralized state feedback control for interconnected systems with application to power systems[☆]

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ABSTRACT

We consider the problem of constructing decentralized state feedback controllers for linear continuous-time systems. Different from existing approaches, where the topology of the controller is fixed *a priori*, the topology of the controller is part of the optimization problem. Structure optimization is done in terms of a minimization of the required feedback links and subject to a predefined bound on the tolerable loss of the achieved $\mathcal{H}_\infty$-performance of the decentralized controller compared to an $\mathcal{H}_\infty$-optimal centralized controller. We develop a computationally efficient formulation of the decentralized control problem by convex relaxations which makes it attractive for practical applications. The proposed design algorithm is applied to design sparse wide area control of a 3-area, 6-machine power system.

1. Introduction

The analysis and control of interconnected systems is one of the big challenges of modern engineering science (see [1]). Examples of such interconnected systems include highly coupled chemical plants, heat exchangers, chemical reaction networks and power generation networks. The constituent parts of interconnected systems are the individual dynamical subsystems, the couplings between these subsystems, the interconnection with the controller and the controller architecture. In a decentralized framework, individual controllers are spatially distributed, and each controller has access to a different subset of local measurements. Decentralized control is often preferred to one centralized controller because less measurement wiring between sensors and controllers is necessary and less information needs to be transmitted [2]. In general, decentralized controller design consists of two different problems: first, the structure of the decentralized controller has to be designed, and second, the controller itself has to be designed. While all this represents a general paradigm for the control of distributed systems and decentralized controller design, there has been a long history within the process control and power systems community focusing on the design of controller architecture and decentralized control. For example the first work on preferable loop pairings [3] was motivated by typical problems in multivariable process control, an alternative approach was for example presented by [4]. Generalizations and other interaction measures can among others be found in [5] and [6]. Recent work on loop pairing and interaction measures based on singular value decomposition and system Gramians include the work of [7,8] and [9]. Interaction measures analyze the underlying structure of an interconnected system and answer therefore the question, how a good structure of the decentralized controller might look like. The design of the decentralized controller itself is independent thereof.

In decentralized controller design, different levels of decentralization can be considered. Fig. 1(a) shows exemplarily a network of three interconnected subsystems Σ_i, $i = 1, 2, 3$ with a centralized controller K that has access to and can influence all the subsystems. In Fig. 1(c), the network is alternatively controlled by three individual controllers K_{ii}, where each controller has only access to its own associated measurements and control inputs. This is a completely decentralized control structure. As an example for an intermediate structure, in Fig. 1(b) additional measured outputs

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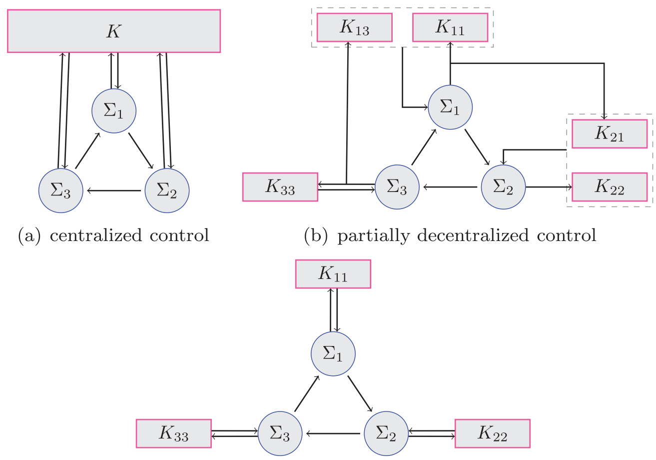

(a) centralized control (b) partially decentralized control (c) completely decentralized control

Fig. 1. Control schemes with different degree of decentralization.

from other subsystems are used to improve the performance of the control loop. This leads to a partially decentralized control structure. Interaction measures as described in the previous paragraph often try to identify the loop pairings, such that a completely decentralized controller as depicted in Fig. 1(c) can be implemented while stability and performance requirements are achieved. However, achievable performance generally decreases for decentralized controllers when compared to centralized ones and sometimes not even the stability of the interconnected system can be guaranteed. It is the goal of this paper to design partially decentralized controllers, with only small $\mathcal{H}_\infty$-performance degradation compared to an optimal centralized controller.

With the advent of convex optimization and the efficient computation of $\mathcal{H}_\infty$- and $\mathcal{H}_2$-optimal controllers, decentralized controller design came again into focus and was considered in [10] and [11], as well as in [12] and [13]. These approaches focus on the design of the $\mathcal{H}_\infty$- or $\mathcal{H}_2$-optimal controller itself and not on the design of the controller architecture. They have the following common features: (i) the structure of the controller to be designed is fixed and restricted to special cases, and plant and controller have to share a common structure; (ii) the structure of the controller has to be specified in advance, that is, they do not consider the problem of structure design but only the design of the controller itself. Considering the problem of controller architecture and controller design itself, a natural question one may raise is whether it is possible to design both the controller structure and the controller itself *simultaneously*. This is especially of interest in the fast growing field of networked control systems with highly interacting subsystems, where it is often not clear, how the controller topology should look like to achieve good performance.

The above discussions motivate the main contribution of this paper. We search for a tradeoff between the number of measurement links, i.e. the degree of decentralization and the achievable $\mathcal{H}_\infty$-performance of the system. We combine control theoretic insight with results from compressed sensing (see e.g. [14,15]) to systematically achieve decentralization with guaranteed system theoretic properties by computationally efficient algorithms. We define a pattern operator to represent the structure of the controller. In this sense, the controller topology is not specified in advance, but considered as an optimization variable. We design state feedback controllers for linear continuous-time interconnected systems, where the number of measurement links is minimized subject to an $\mathcal{H}_\infty$-performance constraint. By means of a system augmentation approach, we first present a novel characterization of the $\mathcal{H}_\infty$-performance of the closed-loop system. Then, the non-convex structure optimization of the pattern operator is relaxed by a convex weighted ℓ_1-minimization, and the resulting problem can be tackled by finding a solution of iterative convex optimization problems. Moreover, we discuss how the initial values of the optimization can be chosen to improve the solvability of the proposed algorithm. The proposed algorithm is applied to design sparse wide-area control for a 3-area, 6-machine power system. Similar results for discrete-time systems can be found in [16] and a preliminary version of the presented results can be found in our previous publication [17]. Joint design of controller structure and controller itself subject to $\mathcal{H}_2$-performance constraint was also reported in [18] and [19].

The remainder of the article is organized as follows. After introducing the mathematical preliminaries in Section 2, the formulation of the decentralized control problem is presented in Section 3. Section 4 is devoted to the design of a decentralized controller for networks of interconnected subsystems. The paper concludes with an example for damping inter-area oscillations in power systems in Section 5 and a summary in Section 6.

2. Mathematical preliminaries

The 0-norm of a vector $x \in \mathbb{R}^n$ is defined as

$$\|x\|_0 = \{\text{number of } x_i | x_i \neq 0\},$$

and corresponds exactly to the number of non-zero entries in x. Despite not being a true norm, it is often referred to one in the literature. A vector is called sparse if its 0-norm is small compared to the dimension of the vector, i.e. if most of its entries are zero. The 0-norm is used in the present context to achieve sparse controller structures. A brief introduction into sparsity measures in general and sparsity promoting optimal control can be found in [18] and [20].

We specify performance in terms of the $\mathcal{H}_\infty$-norm of a system. The $\mathcal{L}_2$-induced norm (or $\mathcal{L}_2$-gain) of a dynamical system $\mathcal{H} : \mathcal{L}_2^n \to \mathcal{L}_2^m$ is defined as

$$\|\mathcal{H}\|_{\mathcal{L}_2-\text{ind}} = \sup_{w \in \mathcal{L}_2 \setminus \{0\}} \frac{\|\mathcal{H}w\|_{\mathcal{L}_2}}{\|w\|_{\mathcal{L}_2}}$$

and corresponds for a linear system $\mathcal{H}$ to the $\mathcal{H}_\infty$-norm $\|H(s)\|_\infty = \sup_\omega \{\overline{\sigma}(H(j\omega)\}$, where $H(s) = C(sI - A)^{-1}B + D$ is a transfer function of the dynamical system $\mathcal{H}$ and $\overline{\sigma}(H(j\omega))$ denotes the largest singular value of H at a fixed frequency ω.

Notational specifications as used in the paper are given next. Given a matrix $M = [m_1 \cdots m_n]$ with $m_i \in \mathbb{R}^n$ being its ith column, we define a vector $\text{vec}(M) \in \mathbb{R}^{nm \times 1}$ by

$$\text{vec}(M) = \begin{bmatrix} m_1^T & \cdots & m_n^T \end{bmatrix}^T.$$

A symmetric and positive definite (resp. positive semi-definite) matrix M is written as $M > 0$ (resp. $M \geq 0$), M^T and M^{-1} denote the transpose and inverse of a matrix M. A (block-) diagonal matrix with elements $m_1, \ldots, m_n$ on the (block-) diagonal is abbreviated as $\text{diag}(m_1, \ldots, m_n)$ or simply $\text{diag}(m_i)$. For $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{m \times n}$, $A \circ B$ denotes the Hadamard product of A and B.

3. Formulation of decentralized control problem

Starting with classical centralized control, the problem of finding a decentralized controller is formulated as a minimization problem over the number of measurement links between subsystems and controllers. This is done subject to an upper bound on the $\mathcal{H}_\infty$-performance degradation between the decentralized control loop and the centralized one. This leads to a block structure in the