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Total value adjustment for European options with two stochastic factors. Mathematical model, analysis and numerical simulation

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ABSTRACT

In the present paper we derive novel (non)linear PDE models for pricing European options and the associated total value adjustment (XVA), when incorporating the counterparty risk. The main innovative aspect is the consideration of stochastic spreads instead of less realistic constant spreads previously used in the literature. For the nonlinear model, a rigorous mathematical analysis based on sectorial differential operators allows to state the existence and uniqueness of a solution. Moreover, for the numerical solution we propose an appropriate set of techniques based on the method of characteristics for time discretization, finite element for spatial discretization and fixed point iteration for the nonlinear terms. Finally, numerical examples illustrate the expected behaviour of the option prices and the total value adjustment.

1. Introduction

Counterparty risk can be understood as the risk to each party of a contract from a future situation in which one of the counterparties cannot live up its contractual obligations. Since the bankrupt of several institutions during the last financial crisis, a relevant effort in quantitative finance research concerns to the consideration of counterparty risk in financial contracts, specially in the pricing of derivatives (see [1–3], for example). As a consequence, different adjustments on the value of the derivative without counterparty risk (hereafter referred as risk-free derivative) are being included in the derivative pricing. For example, the credit value adjustment (CVA) refers to the variation on the price of a contract due to the possibility of default of one (or both) of the counterparties. Adjustments on debit (DVA) and funding (FVA) are also important issues included in the so called total value adjustment (XVA). The XVA incorporates the sum of all the adjustments related to counterparty risk.

Among the methodologies to include the ingredients involved in the XVA, one of them leads to partial differential equation (PDE) formulations. More precisely, this approach starts from the works by Piterbarg [4], where funding costs are included, and Burgard and Kjaer [5], that considers funding costs and bilateral counterparty risk. In both works, for the case of a European style option the PDE formulation is based on the use of appropriate hedging arguments combined with Itô's lemma for jump–diffusion processes [6]. This is the approach we follow here. Recently, in [7] a one factor model to price the adjustments associated to European and American options has been analysed and numerically solved. In particular,

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funding value adjustment (FVA), debit value adjustment (DVA) and credit value adjustment (CVA) have been considered. Furthermore, the model in [7] is extended to incorporate the collateral value adjustment(CollVA), in case that a collateral is used to guarantee the obligations related to the options contract. In [7], constant default intensities for both counterparties have been considered, so that a model depending on just one underlying stochastic factor (the underlying asset) is posed.

However, counterparties default intensities do not always exhibit constant behaviours. In a general framework, intensities might follow a stochastic process [8]. In the present work we focus on the European options pricing and the corresponding XVA adjustments when stochastic intensities are assumed. More precisely, we state PDE models for the derivative value from the point of view of an investor, when the trade takes place between two counterparties: an investor and a hedger. If we consider stochastic intensities of default for both counterparties then a model with three stochastic factors is obtained [8]. Our approach is based on the same framework and assumptions as in [8], although with the additional hypothesis of a zero default intensity for a hedger, thus leading to a two stochastic factors model. However, a case of an additional stochastic intensity only increases the spatial dimension of the PDE, so that we trust that the theoretical analysis and numerical methods here used can be extended to this more general setting.

As in [8], we include all the components in the pricing of uncollateralized derivatives with counterparty risk, with the following assumptions:

- The price of a derivative should reflect all of its hedging costs.
- Since in a high percentage of uncollateralized transactions the presence of an investor (risk taker) and a hedger (risk hedger) is implied, the price of the derivative should just reflect the hedging costs transmitted by the hedger.
- The hedger will only be willing to hedge the fluctuations in the price of the derivative that he will experience while not having defaulted.
- There is neither CVA nor FVA to be made to fully collateralized derivatives (with continuous collateral margining in cash, symmetrical collateral mechanism and no threshold, minimum transfer amount, etc.).

Moreover, we will consider the following market assumptions:

- There is a liquid CDS (credit default swap) curve for the investor.
- There is a liquid curve of bonds issued by the hedger.
- Continuous hedging, unlimited liquidity, no bid-offer spreads, no trading costs.
- Recovery rates are either deterministic or there are recovery locks available so that recovery risk is not a concern,

as well as the following model assumptions:

- Only the investor is defaultable.
- The underlying asset follows a diffusion process under the real world measure.
- The underlying asset of a derivative is unaffected by a default event of the investor.
- The investor credit spread is stochastic and follows a diffusion process correlated with the asset price under the real world measure.

Keeping in mind these assumptions, in the present paper we state a PDE formulation by means of suitable hedging arguments and the use of Itô's Lemma for jump–diffusion processes [6]. After arguing the hedging strategy, different linear or nonlinear PDEs arise depending on the choice of the mark-to-market value at the default. If the mark-to-market value is equal to the risk-free derivative price then a linear PDE involving the riskless derivative price is obtained. Alternatively, if the mark-to-market value is equal to the risky derivative price, then a nonlinear PDE arises. For the nonlinear PDE formulation we develop the mathematical analysis of the model to obtain existence and uniqueness of a solution in the appropriate functional space on a bounded domain. For this purpose, we use the tools of nonlinear parabolic PDEs involving sectorial operators [9].

In addition, we propose a set of numerical methods to solve the PDEs for both choices of the mark-to-market value. First, we truncate the unbounded domain and formulate suitable boundary conditions at the boundaries of the resulting bounded domain, following some ideas in [10]. Next, we propose a time discretization based on the method of characteristics combined with a finite element discretization in the asset and spread variables. The method of characteristics has been proposed in [11] in the context of fluid mechanics problems and used in finance in [12] for vanilla options and in [10,13] for Asian options or in [14] for pension plans. For the nonlinear PDE a fixed point iteration algorithm is additionally proposed.

The plan of the article is as follows. In Section 2 we propose the mathematical model. Section 3 is devoted to the mathematical analysis of the nonlinear PDE problem that models the price of the XVA. Furthermore, we prove the existence and uniqueness of solution. In Section 4 we describe numerical methods we propose to compute a solution of our models. In Section 5, we show and discuss the numerical results for some illustrative examples. Finally, Section 6 contains some conclusions.

2. Two stochastic factors models

In this section, we obtain the models for European options and their associated XVA pricing when the counterparty risk and funding costs are taken into account. The main difference with the one factor model presented in [7] comes from the