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A boundary-type meshless solver for transient heat conduction analysis of slender functionally graded materials with exponential variations

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ABSTRACT

This study presents a parallel meshless solver for transient heat conduction analysis of slender functionally graded materials (FGMs) with exponential variations. In the present parallel meshless solver, a strong-form boundary collocation method, the boundary knot method (BKM), in conjunction with Laplace transform is implemented to solve the heat conduction equations of slender FGMs with exponential variations. This method is mathematically simple, easy-to-parallel, meshless, and without domain discretization. However, two ill-posed issues, the ill-conditioning dense BKM matrix and numerical inverse Laplace transform process, may lead to incorrect numerical results. Here the extended precision arithmetic (EPA) and the domain decomposition method (DDM) have been adopted to alleviate the effect of these two ill-posed issues on numerical efficiency of the present method. Then the parallel algorithm has been employed to significantly reduce the computational cost and enhance the computational capacity for the FGM structures with larger length-width ratio. To demonstrate the effectiveness of the present parallel meshless solver for transient heat conduction analysis, several benchmark examples are considered under slender FGMs with exponential variations. The present results are compared with the analytical solutions, the conventional boundary knot method and COMSOL simulation.

1. Introduction

Functionally graded materials (FGMs) [1] usually consist of two constituent materials, where their microstructure varies from one material to another with a specified gradient. Due to their superior material properties, FGMs offer benefits for thermal barrier coating systems for aircraft and power generation gas turbines. For the optimal design of the thermal barrier coating systems with functionally graded materials, it is necessary to investigate the thermal behavior under the slender FGMs. Analytical methods are usually restricted to simple physical domains and boundary conditions. Therefore, in the past decades, extensive studies have been carried out on developing numerical methods for analyzing thermal behaviors of FGMs, for example, the finite element method (FEM) [2–4], the boundary element method (BEM) [5,6], the meshless local Petrov–Galerkin method (MLPG) [7,8], the method of fundamental solution (MFS) [9,10] and the boundary knot method

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(BKM) [11,12], to mention just a few. However, the conventional FEM is inefficient for handling materials whose physical property varies continuously, and requires very dense mesh to obtain the acceptable results for slender structure with large aspect ratio. The BEM needs to treat the singular and near-singular integrals [13,14], which is mathematically complex and requires extensive computational resources. Unlike the FEM and the BEM, the meshless methods do not require the mesh generation, and which are widely used to heat conduction analysis under the FGMs. Among the above-mentioned meshless methods, the MLPG belongs to the category of weak-formulation, and the MFS and the BKM belong to the category of strong-formulation.

Since their inherent merits of easy-to-programming and integration-free, this study focuses on the strong-form meshless methods. In the MFS one has to construct a fictitious boundary [15–18] outside the physical domain to avoid the singularities of fundamental solutions. However, selecting the appropriate fictitious boundary plays a vital role for the accuracy and reliability of the MFS solution. To avoid the singularities of fundamental solutions and the controversial fictitious boundary in the MFS, an alternative approach, the boundary knot method, has been proposed by Chen and Tanaka [19]. Therefore, the boundary knots can be placed on the physical boundary. In the BKM one employs the nonsingular radial basis function (RBF) general solution instead of the singular fundamental solution, and thus the boundary nodes are placed on the physical boundary.

On the other hand, the BKM has been employed to deal with transient heat conduction problems through three different approaches: (1) time-dependent basis function method [20], one needs to derive the corresponding nonsingular time-dependent general solution as a priori to satisfy the transient heat conduction equation and then solve it directly; (2) time stepping method [21,22], one transforms the transient heat conduction problem into time-independent inhomogeneous problem then introduces some additional particular techniques to solve this inhomogeneous problem; (3) Laplace transform technique [6], one uses the Laplace transformation of governing equation to eliminate the time derivative leading to a time-independent heat conduction equations in Laplace space, which can be solved by boundary meshless methods, and then employ numerical Laplace inversion scheme to invert the Laplace space solutions back into the time-dependent solutions. The Laplace transform technique does not require time marching, and thus avoids the effect of the time step on numerical accuracy.

Therefore, this study will use the BKM in conjunction with Laplace transformation (LTBKM) for transient heat conduction analysis of slender FGMs. However, in the BKM the ill-condition matrix occurs with a medium number of boundary nodes. Moreover, the larger aspect-ratio the slender FGM has, the more ill-conditioning the BKM matrix is. This may jeopardize the numerical accuracy. In addition, numerical inverse Laplace transform is an ill-posed problem, the truncation error magnification phenomena appears in the inversion process and results in a loss of numerical accuracy. There two drawbacks limit its application for slender FGMs with larger aspect-ratio.

To alleviate the effect of the above-mentioned two ill-posed issues, the extended precision arithmetic (EPA) [23,24], the domain decomposition method (DDM) [25] and the parallel computation have been adopted in the present LTBKM framework. Then the proposed parallel meshless solver is applied to heat conduction analysis in slender FGMs.

A brief outline of the paper is as follows. Section 2 introduces the parallel meshless solver including the boundary knot method in conjunction with Laplace transformation, the extended precision arithmetic (EPA), the domain decomposition method (DDM) and the parallel computation. Section 3 investigates the numerical efficiency of the proposed approaches through several typical examples. Finally, some conclusions are presented in Section 4.

2. Methodology

2.1. Heat conduction model of slender FGMs

Consider two-dimensional transient heat conduction problem in slender functionally graded materials as shown in Fig. 1, occupying a 2D slender region $\Omega \subset \Re^2$ bounded by its boundary Γ, and in the absence of heat sources. The governing equation is stated as

$$\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(K_{ij}(\mathbf{x})\frac{\partial u(\mathbf{x},t)}{\partial x_j}\right) = \rho(\mathbf{x})\, c(\mathbf{x})\frac{\partial u(\mathbf{x},t)}{\partial t}, \quad \mathbf{x}=(x_1,x_2)\in\Omega \tag{1}$$

with the boundary and initial conditions:

Dirichlet/Essential condition

$$u(\mathbf{x},t) = g_1(\mathbf{x},t), \ \ \mathbf{x}=(x_1,x_2)\in\Gamma_k=\Gamma_D, \ \ k=1,2,3,4. \tag{2a}$$

Neumann/Natural condition

$$q(\mathbf{x},t) = -\sum_{i,j=1}^{2} K_{ij}\frac{\partial u(\mathbf{x},t)}{\partial x_j}n_i = g_2(\mathbf{x},t), \mathbf{x}=(x_1,x_2)\in\Gamma_k=\Gamma_N, \ \ k=1,2,3,4. \tag{2b}$$