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Fully nonlinear capillary–gravity solitary waves under a tangential electric field, Part II: Dynamics

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ABSTRACT

The stability and dynamics of two-dimensional fully nonlinear capillary–gravity solitary waves are studied when a uniform electric field is applied in the direction parallel to the undisturbed free surface of a dielectric fluid. For simplicity, we assume that the permittivity of the gas above the fluid is much smaller, therefore the two-layer problem can be reduced to be a one-layer one. The existence of fully localized solitary waves in deep water has been thoroughly studied in our previous work (Tao and Guo, 2014) via weakly nonlinear normal form analysis and fully nonlinear numerical computation. In the present paper, the stability and dynamics of the obtained solitary waves are investigated based on the time-dependent conformal map technique. Similar to the nonelectric capillary–gravity solitary waves, all depression waves, together with elevation waves featuring two big troughs connected by a small dimple, are found to be stable. Furthermore, head-on and overtaking collisions between stable solitary waves are studied via a numerical time integration.

1. Introduction

Gravity-capillary surface waves on an electrified liquid exposed to an external electric field enjoy a wide usage in physics, biology and industry, such as vacuum discharge, coating process, electrowetting, biology membranes, etc. Usually, the external electric field is posed in tangential or normal direction relative to the undisturbed fluid surface. The normal electric field tends to destabilize the system, and a typical example is the Taylor cone experiment [1]. When a small volume of a conducting fluid is under a normal electric field, the instability is induced and the shape of the liquid starts to deform which leads to the Taylor cone formation. As the voltage is above a certain threshold, the tip of the cone can emit a jet of liquid. Conducting fluids subject to the normal electric fields and related problems have been studied by many authors, and the readers are referred to [2–12] and the references therein. On the contrary, the tangential electric field has a stabilizing effect on the surface of the electrified fluid. It can provide linear and nonlinear dispersive contributions to the system [13–17], delay the formation of the film rupture [18], and even suppress the Rayleigh–Taylor instabilities [19,20]. In this paper, we consider the later case where the tangential electric field is present and competes with the effects of gravity and surface tension.

Nonelectric gravity–capillary waves have received considerable attention in the past few decades due to their applications in the wind–ocean coupling system. On the numerical side, of note is the work by Longuet-Higgins [21] who first computed symmetric gravity–capillary solitary waves with a trough at their center (i.e. depression waves), Vanden-Broeck & Dias [22] who found another branch of symmetric solutions with a positive free-surface hump at the center (i.e. elevation waves), Wang et al. [23] who found asymmetric gravity–capillary solitary waves in the full Euler equations, and Milewski et al. [24] and Wang [25] who thoroughly investigated the stability and dynamics of these solutions.

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When a tangential electric field is present, Parageorgiou & Vanden-Broeck [14,15] studied the electrocapillary waves, where progressive waves with a permanent form were computed in the full Euler equations via the boundary integral method with an arclength parameterization, and the dynamics were investigated by novel long wave model equations. Barannyk et al. [19] and Cimpeanu et al. used tangential electric fields to stabilize the Rayleigh–Taylor instability for inviscid and viscous fluids respectively. Tilley et al. [18] derived a set of long wave models to investigate the stability of a thin liquid film when a uniform electric field was applied in a direction parallel to the unperturbed fluid interfaces. They showed that the presence of the electric field causes a nonlinear stabilization of the flow in that it delays singularity formation. Zubarev and his collaborator in [16,17] studied the similar problem using another approximation. He considered the gas–fluid or vacuum–fluid interface so as to make the assumption that the permittivity of the fluid was much larger compared to that of the gas (the permittivities for water and air are 80 and 1 respectively). Therefore, to the leading order, the two-layer problem can be reduced to a one-layer one, namely, we can solve the electric voltages in the air and fluid separately. With Zubarev's approximation, Tao & Guo [26] numerically computed the electrified gravity–capillary solitary waves subject to a tangential electric field based on a hodograph transformation for the full Euler equations. In this paper, we continue to study the stability and dynamics of solitary wave solutions found in [26], and the time-dependent conformal map technique is used for an inviscid, incompressible and irrotational dielectric fluid where the distinct physical effects of gravity, surface tension and electrically induced forces are all taken into account.

The rest of the paper is organized as follows. In Section 2, the governing equations are described and simplified by making the assumption that the permittivity of the fluid is much larger than that of the air. In Section 3 the numerical method for computing fully nonlinear, unsteady gravity–capillary waves in the presence of a tangential electric field is derived. Then the main numerical results, including the stability and dynamics of solitary waves, are then followed. The concluding remarks are given in Section 4.

2. Governing equations

2.1. Formulation and simplification

We consider a two-dimensional inviscid, incompressible fluid flowing irrotationally. The typical wavelength of gravity–capillary waves in water is about 1–2 cm, therefore throughout the paper it is physically reasonable to assume that the fluid is of infinite depth. We introduce the coordinates so that x is the horizontal axis, y the vertical coordinate pointing upwards and t time. We denote by $y = \eta(x, t)$ the free surface of the fluid which reduces to $y = 0$ if there is no perturbation. The fluid is taken to be perfect dielectric with constant permittivity $\widetilde{\epsilon}$. A uniform electric field, which is parallel to the undisturbed free surface and of the strength E_0 in the far field, is applied throughout the entire space. Due to the irrotationality of the fluid, we introduce a potential function $\phi(x, y, t)$ such that the velocity of the fluid can be expressed as $\nabla\phi$. Following [14,15], we can introduce the voltage potentials $\widetilde{V}(x, y, t)$ and $\widetilde{V}^+(x, y, t)$ for the lower-layer fluid and upper-layer gas respectively. It follows that all the field equations satisfy the Laplace equation,

$$\begin{cases} \phi_{xx} + \phi_{yy} = \widetilde{V}_{xx} + \widetilde{V}_{yy} = 0\,, & \text{for } -\infty < y < \eta\,, \\ \widetilde{V}^+_{xx} + \widetilde{V}^+_{yy} = 0\,, & \text{for } \eta < y < +\infty\,. \end{cases} \tag{2.1}$$

The voltage potential is continuous through the fluid–gas interface $y = \eta(x, t)$, therefore

$$\widetilde{V} = \widetilde{V}^+\,, \tag{2.2}$$

while the normal stress on the boundary satisfies

$$\widetilde{\epsilon}\,(\widetilde{V}_y - \eta_x\widetilde{V}_x) = \epsilon^+(\widetilde{V}^+_y - \eta_x\widetilde{V}^+_x)\,, \tag{2.3}$$

where $\widetilde{\epsilon}$ and ϵ^+ are the electric permittivities of the fluid and gas (or vacuum) respectively (see, for example, [27,14,15,26]). For the motion of the free surface, the kinematic and dynamic boundary conditions read

$$\eta_t = \phi_y - \eta_x\phi_x\,, \tag{2.4}$$

$$\rho\Big(\phi_t + \frac{1}{2}\big|\nabla\phi\big|^2 + g\eta\Big) + \frac{\widetilde{\epsilon}}{2}\,\frac{\eta_x^2 - 1}{\eta_x^2 + 1}\Big[\big(\widetilde{V}_x\big)^2 - \big(\widetilde{V}_y\big)^2\Big] - \frac{2\widetilde{\epsilon}\eta_x}{\eta_x^2 + 1}\widetilde{V}_x\widetilde{V}_y$$
$$- \frac{\epsilon^+}{2}\,\frac{\eta_x^2 - 1}{\eta_x^2 + 1}\Big[\big(\widetilde{V}^+_x\big)^2 - \big(\widetilde{V}^+_y\big)^2\Big] + \frac{2\epsilon^+\eta_x}{\eta_x^2 + 1}\widetilde{V}^+_x\widetilde{V}^+_y = \sigma\,\frac{\eta_{xx}}{(1 + \eta_x^2)^{3/2}}\,, \tag{2.5}$$

where ρ is the fluid density, g represents the acceleration due to gravity, and σ is the surface tension coefficient. In order to complete the system, boundary conditions in the far field are required:

$$\phi_y \to 0, \qquad \text{as } y \to -\infty\,, \tag{2.6}$$

$$\widetilde{V}_x \to E_0, \quad \widetilde{V}^+_x \to E_0, \quad \text{as } x^2 + y^2 \to \infty\,. \tag{2.7}$$

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