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Closed form solutions for coupled nonlinear Maccari system

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ABSTRACT

The exploration of closed form solutions for nonlinear partial differential equations (NLPDEs) is being an attractive subject in the different branches of mathematical and physical sciences. In this paper, we have applied extended Exp-function method to calculate closed form solutions for NLPDEs; such as the nonlinear Maccari system which is very significant in engineering and mathematical physics. Plentiful closed form solutions with arbitrary parameters are successfully obtained by this method. It is shown that the obtained solutions are more general and fresh and can be helpful to analyze the NLPDEs in mathematical physics and engineering problems.

1. Introduction

Most of the phenomena in the real world are described by nonlinear evolution equations (NLEEs). The nonlinear processes are most challenging and difficult issue for its nonlinear characteristics and not easy to control the system tersely changes because of some small changes of valid parameters. Thus the subject turns more complex and hence conclusive solution of NLEEs is desired. In particular soliton solutions are extremely remarkable in the study of the models arising from scientific fields; for instance, the wave phenomena observed in the fluid mechanics, plasma physics, nuclear physics, high-energy physics, optical fibers, solid state physics etc. Since closed form solution of nonlinear partial differential equations symbolically and graphically demonstrate connection of inner mechanism of many complex nonlinear phenomena, therefore closed form solutions of NLEEs cooperate an essential role in order to better understanding the qualitative structures of many complex processes and phenomena in the mentioned areas of natural sciences. As a result, the closed form solutions of NLEEs have been examined by many researchers who are interested in nonlinear phenomena which exist in all fields including either the scientific works or engineering fields. Noteworthy developments have been done for examining the closed form solutions of NLEEs in recent years. Many influential and effective approaches have been proven to handle the NLEEs, such as, First integration method [1], Exp-function method [2,3], Modified Exp-function [4], Tanh-function method [5], Hirota's bilinear transformation method [6], F-expansion method [7], Sine–Cosine method [8], modified simple equation method [9], Exp($-\Phi(\eta)$)-expansion method [10,11], (G'/G) expansion method [12,13], Novel (G'/G)-expansion method [14,15], Alternative (G'/G)-expansion method [16], Enhanced (G'/G)-expansion method [17] etc.

By using an asymptotically exact reduction method which is based on two different approaches called the Fourier expansion and spatio-temporal rescaling, Maccari [18] derived the $(2+1)$-dimensional system from the Kadomtsev–Petviashvili equation. By executing the reduction technique to the Lax pair of Kadomtsev–Petviashvili equation, the integrability property is evidently indicated by displaying the corresponding Lax pair that is obtained.

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Now, consider the following Maccari system:

$$\begin{aligned} & ih_t + h_{xx} + hw = 0, \\ & w_t + w_y + \left(|h|^2\right)_x = 0, \end{aligned} \tag{1}$$

where h and w are dependent variables representing the complex scalar and real scalar fields. The independent spatial variables are x and y, whereas t is the time-based variable [19].

In many fields of science and engineering, to define the motion of the quarantined waves contained in a small part of space, such as quantum field theory, hydrodynamics, to describe the behavior of the sonic Langmuir solitons in plasma physics, and in nonlinear optics [20], Maccari system is frequently used.

Many researchers tried to calculate exact solutions of Eq. (1) by means of different approaches [21–26] but extended Exp-function method had never been applied on it before.

2. The method

Consider the general form of a NLPDE as follows:

$$P\left(h,\ h_x, h_y,\ h_t, h_{xx}, \ldots\right) = 0, \tag{2}$$

where P is a polynomial in its arguments, contains both the higher-order derivatives and the nonlinear terms of h.

The phases of construction of the proposed scheme are as follows:

Phase 1: The conversion

$$h\ (x, y, t) = h\left(\rho\right), \qquad \rho = k(x + y + V\, t), \tag{3}$$

transforms Eq. (2) into a nonlinear ordinary differential equation (ODE)

$$M\ \left(h, h', h'', h''', \ldots\right) = 0, \tag{4}$$

in which primes stand for the ordinary derivatives with respect to ρ and k, V are the wave number and velocity of the wave respectively.

Phase 2. Eq. (4) can be integrated if possible, yields constant(s) of integration.

Phase 3: The assumed wave solution of Eq. (4) is represented as:

$$h(\rho) = \frac{\sum_{i=0}^{l_1} \Omega_i [\exp\left(-\varphi\left(\rho\right)\right)]^i}{\sum_{j=0}^{l_2} \phi_j [\exp\left(-\varphi\left(\rho\right)\right)]^j} = \frac{\Omega_0 + \Omega_1 \exp\left(-\varphi\left(\rho\right)\right) + \cdots + \Omega_{l_1} \exp\left(l_1\left(-\phi\left(\rho\right)\right)\right)}{\phi_0 + \phi_1 \exp\left(-\varphi\left(\rho\right)\right) + \cdots + \phi_{l_2} \exp\left(l_2\left(-\phi\left(\rho\right)\right)\right)}, \tag{5}$$

where Ω_i, ϕ_j, $(0 \le i \le l_1, 0 \le j \le l_2)$ are constants to be resolute later, such that $\Omega_{l_1} \neq 0$, $\phi_{l_2} \neq 0$, and $\varphi = \varphi\left(\rho\right)$ satisfies the nonlinear ordinary differential equation;

$$\varphi'\left(\rho\right) = \exp\left(-\varphi\left(\rho\right)\right) + a\ \exp\left(\varphi\left(\rho\right)\right) + b. \tag{6}$$

The solutions of Eq. (6) are as follows [27,28]:

Family 1: When $a \neq 0$, $b^2 - 4a > 0$,

$$\varphi\left(\rho\right) = \ln\left[\frac{-\sqrt{b^2 - 4a}}{2a}\tanh\left(\frac{\sqrt{b^2 - 4a}}{2}\left(\rho + c\right)\right) - \frac{b}{2a}\right], \tag{7}$$

Family 2: When $a \neq 0$, $b^2 - 4a < 0$,

$$\varphi\left(\rho\right) = \ln\left[\frac{\sqrt{-b^2 + 4a}}{2a}\tan\left(\frac{\sqrt{-b^2 + 4a}}{2}\left(\rho + c\right)\right) - \frac{b}{2a}\right], \tag{8}$$

Family 3: When $a = 0$, $b \neq 0$, and $b^2 - 4a > 0$,

$$\varphi\left(\rho\right) = -\ln\left(\frac{b}{\exp\left(b\left(\rho + c\right)\right) - 1}\right), \tag{9}$$

Family 4: When $a \neq 0$, $b \neq 0$, and $b^2 - 4a = 0$,

$$\varphi\left(\rho\right) = \ln\left(-\frac{2\, b\left(\rho + c\right) + 4}{b^2\left(\rho + c\right)}\right), \tag{10}$$

Family 5: When $a = 0$, $b = 0$, and $b^2 - 4a = 0$,

$$\varphi\left(\rho\right) = \ln\left(\rho + c\right), \tag{11}$$