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Boundedness in a 2D chemotaxis-Stokes system with general sensitivity and nonlinear diffusion

Yilong Wang

School of Sciences, Southwest Petroleum University, Chengdu 610500, China
Institute of Nonlinear Dynamics, Southwest Petroleum University, Chengdu 610500, China

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ABSTRACT

This paper deals with the following chemotaxis-Stokes system

$$\begin{cases} n_t + u\cdot\nabla n = \Delta n^m - \nabla\cdot(nS(x,n,c)\cdot\nabla c), & x\in\Omega,\ t>0,\\ c_t + u\cdot\nabla c = \Delta c - nf(c), & x\in\Omega,\ t>0,\\ u_t = \Delta u + \nabla P + n\nabla\phi, & x\in\Omega,\ t>0,\\ \nabla\cdot u = 0, & x\in\Omega,\ t>0 \end{cases}$$

in a bounded domain $\Omega\subset\mathbb{R}^2$ with smooth boundary, $\phi\in W^{1,\infty}(\Omega)$, f and S are given sufficiently smooth functions with values in $[0,\infty)$ and $\mathbb{R}^{2\times2}$, respectively. Here S satisfies $|S(x,n,c)|<S_0(c)n^\alpha$ with $\alpha\geq0$ and some nondecreasing nonnegative function S_0. It is showed that when $m>1+\alpha$ and $\alpha\geq0$, the corresponding system possesses a global bounded weak solution for any sufficiently regular initial data $(n_0,\ c_0,\ u_0)$ satisfying $n_0\geq0$ and $c_0\geq0$.

1. Introduction

We consider the following initial–boundary value problem of a degenerate chemotaxis-Stokes model

$$\begin{cases} n_t + u\cdot\nabla n = \Delta n^m - \nabla\cdot(nS(x,n,c)\cdot\nabla c) & x\in\Omega,\ t>0,\\ c_t + u\cdot\nabla c = \Delta c - nf(c), & x\in\Omega,\ t>0,\\ u_t = \Delta u + \nabla P + n\nabla\phi, & x\in\Omega,\ t>0,\\ \nabla\cdot u = 0, & x\in\Omega,\ t>0,\\ (\nabla n^m - nS(x,n,c)\cdot\nabla c)\cdot\nu = \nabla c\cdot\nu = 0,\ u=0 & x\in\partial\Omega,\ t>0,\\ n(x,0)=n_0(x),\ c(x,0)=c_0(x),\ u(x,0)=u_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^2$ is a bounded domain with smooth boundary, $m>1$, the known function $\phi\in W^{1,\infty}(\Omega)$ denotes the gravitational potential, ν represents the outward normal unit vector to $\partial\Omega$. The unknown functions $n(x,t)$, $c(x,t)$, $u(x,t)$ and $P(x,t)$ denote the density of the bacteria, the concentration of the oxygen, the velocity of fluid and the associated pressure, respectively. Here $S(x,n,c)$ is a chemotactic sensitivity tensor satisfying

$$S\in C^2\left(\bar\Omega\times[0,\infty)^2;\ \mathbb{R}^{2\times2}\right) \tag{1.2}$$

and

$$|S(x,n,c)|\leq S_0(c)n^\alpha \qquad \text{for all}\quad (x,n,c)\in\bar\Omega\times[0,\infty)^2 \tag{1.3}$$

E-mail address: wangelongelone@163.com.

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with some nondecreasing function $S_0 : [0, \infty) \to [0, \infty)$ and $\alpha \geq 0$. $f(c)$ denotes the consumption rate of the oxygen by the cells and

$$f \in C^1([0, \infty)) \quad \text{satisfies } f(0) = 0 \text{ and } f(c) > 0 \text{ for all } c > 0. \tag{1.4}$$

The initial data satisfy

$$\begin{cases} n_0 \in C^0(\bar{\Omega}), \qquad n_0 \geq 0 \text{ in } \bar{\Omega}, \\ c_0 \in W^{1,\infty}(\Omega), \quad c_0 \geq 0 \text{ in } \bar{\Omega}, \\ u_0 \in D(A_r^\beta) \quad \text{for some } \beta \in (\frac{1}{2}, 1) \text{ and } r \in (1, \infty), \end{cases} \tag{1.5}$$

where A_r stands for the Stokes operator with domain $D(A_r) := W^{2,r}(\Omega) \bigcap W_0^{1,r}(\Omega) \bigcap L^r_\sigma(\Omega)$ [1]. Here $L^r_\sigma := \{\varphi \in L^r(\Omega) \mid \nabla \cdot \varphi = 0\}$ for $r \in (1, \infty)$. Problem (1.1) describes the motion of oxygen-taxis bacteria swimming in an incompressible viscous fluid.

To model the mutual interaction between cells and the surrounding water, Tuval et al. [2] proposed the following chemotaxis-(Navier)–Stokes system

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \nabla \cdot (nS(c)\nabla c) & x \in \Omega,\ t > 0, \\ c_t + u \cdot \nabla c = \Delta c - nf(c), & x \in \Omega,\ t > 0, \\ u_t + \kappa(u \cdot \nabla)u = \Delta u + \nabla P + n\nabla\phi, & x \in \Omega,\ t > 0, \\ \nabla \cdot u = 0, & x \in \Omega,\ t > 0, \end{cases} \tag{1.6}$$

where $\kappa \in \{0, 1\}$ and $S(c)$ is a scalar function measuring the chemotactic sensitivity. This system shows that the motion of individual cells is chemotactically bias by concentration gradients of dissolved oxygen which they consume, and that moreover cells and oxygen are transported through water via convection, and that the swimming cells affect the fluid motion through buoyant forces. The scholars in [3] extended system (1.6) by considering nonlinear diffusion. For system (1.6) and its variants (with nonlinear diffusion), there exist some literatures on global existence, boundedness and large time behavior for the bounded convex domains or the whole space (see [4–24] and the references therein). In the above literatures, there exist various functional frameworks, which take the key role in global existence theories as well as the large time asymptotics of solutions.

The environment for the bacteria is generally more complicated and other external forces should be considered, such as gravity. Recent experimental findings and corresponding modeling approaches in [25] suggested that chemotactic migration is not directed to the gradient of the chemical substance but with a rotation, especially near the physical boundary of the domain. And they suggested that the chemotactic sensitivity should be considered as a tensor, which may have nontrivial off-diagonal entries. Thus, system (1.6) is generalized to the following system

$$\begin{cases} n_t + u \cdot \nabla n = \Delta n - \nabla \cdot (nS(x, n, c) \cdot \nabla c) & x \in \Omega,\ t > 0, \\ c_t + u \cdot \nabla c = \Delta c - nf(c), & x \in \Omega,\ t > 0, \\ u_t + \kappa(u \cdot \nabla)u = \Delta u + \nabla P + n\nabla\phi, & x \in \Omega,\ t > 0, \\ \nabla \cdot u = 0, & x \in \Omega,\ t > 0, \end{cases} \tag{1.7}$$

where $S(x, n, c)$ is a tensor-valued function, which indicates the rotational effect.

There are fewer results on global existence for system (1.7) than system (1.6), since system (1.7) seems to lack a useful energy-type functional unlike system (1.6). For the case that $S(x, n, c)$ satisfies (1.2) and (1.3) with $\alpha = 0$, Li et al. [26] proved existence of a global bounded classical solution under smallness on c_0 for a two dimensional fluid-free subcase of system (1.7) in a bounded domain; and there exists at least one global generalized solution to this fluid-free chemotaxis system (1.7) for any large initial data and any choice of $N \geq 1$, but its regularity properties may be rather poor [27]; more recently, Winkler [28] constructed certain global mass-preserving generalized solutions to an associated initial–boundary value problem (1.7) with $\kappa = 0$ in planar convex domains with smooth boundary, provided that the initial data and the parameter functions S, f and ϕ are sufficiently smooth, and that S satisfies (1.2) and (1.3) with $\alpha = 0$ and f is nonnegative with $f(0) = 0$. When the initial data satisfy certain smallness conditions and S satisfies (1.2) and (1.3) with $\alpha = 0$ and $f(c) = c$, Cao et al. [29,30] showed that chemotaxis(-Navier)–Stokes system (1.7) has global classical solutions in a bounded domain $\Omega \subset \mathbb{R}^N$, $N \in \{2, 3\}$, and decay properties of these solutions are given. When tensor-valued function S satisfies saturation effects at large cell densities, that is, $|S| \leq C(1 + n)^{-\alpha}$ $(\alpha \geq 0)$ with some positive constant C, Wang et al. [31,32] showed that when $\alpha > \frac{1}{6}$, chemotaxis-stokes system (1.7) has a global bounded classical solution in a bounded domain $\Omega \subset \mathbb{R}^3$.

For the case that diffusion is nonlinear and enhanced at large densities, that is, Δn is replaced by Δn^m $(m > 1)$, there exists a global bounded weak solution to the fluid-free subcase of (1.7) for any initial data in a bounded domain $\Omega \subset \mathbb{R}^2$ [33]; Wang [34,35] showed that the fluid-free subcase of (1.7) possesses a global bounded weak solution for general tensor-valued function S and any initial data in a bounded domain $\Omega \subset \mathbb{R}^N (N \geq 2)$. When $\kappa = 0$ and $m > \frac{7}{6}$, system (1.7) admits global bounded weak solutions in a convex bounded domain $\Omega \subset \mathbb{R}^3$ [36]; when $\kappa = 1$ and $m > 1$, system (1.7) admits a global bounded weak solution in a bounded domain $\Omega \subset \mathbb{R}^2$ [37]. When tensor-valued function S satisfies saturation effects at large cell densities, that is, $|S| \leq C(1 + n)^{-\alpha}$ $(\alpha \geq 0)$ with some positive constant C, Wang and

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