ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

The effects of technological shocks in an optimal goodwill model with a random product life cycle

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ARTICLE INFO

Article history:
Received 1 August 2016
Received in revised form 16 May 2018
Accepted 27 May 2018
Available online xxxx

Keywords:
Lotka–Sharp–McKendrick PDE
Random product life cycle
Existence of an optimal solution
Product goodwill
Marketing strategy
Consumer recommendation

ABSTRACT

We consider the optimal goodwill control problem in a segmented market where the length of the product life cycle is affected by unpredictable technological turbulence. In order to maximize profit over a random time horizon, a company controls the marketing efforts directed to each market segment. Assuming an exponential distribution for the product life cycle, we modify the optimal goodwill model into the infinite time horizon control problem. Based on the semigroup approach, we prove the existence and uniqueness of the optimal solution. We formulate optimality conditions for the problem and we prove the existence of a stationary long-run equilibrium. Next, we construct a numerical algorithm to find the optimal solution. Finally, we examine several scenarios of optimal marketing strategies.

1. Introduction

Product goodwill is generated by long-term customer–company relationship (see Liu [1]). Researchers and business practitioners agree that a longitudinal relationship with customers produces a number of important benefits for a company such as decreasing price sensitiveness, increasing profit or competitiveness (see Venetis and Ghauri [2]). Scholars (e.g., Eggert et al. [3], Smith and Colgate [4]) recognize many activities which can strengthen this type of relationship. They include, among others, product quality, service support, delivery performance, supplier know-how, time-to-market and personal interaction. Therefore, models describing this phenomenon as a long-term process, usually, are considered with an infinite planning horizon (see the seminal paper by Nerlove and Arrow [5], Wang et al. [6]). However, rapid technological changes cause modifications of the products attributes and functionality, so that after some upgrades, it becomes a completely different product (see Gurnani [7]). Therefore recently models describing management of resources on infinite horizon are reformulated to obtain models with a finite planning horizon (see Taheri-Tolgari et al. [8], Górajski and Machowska [9], Reddy et al., [10]). However, significant technological innovations, due to their nature, appear on the market with unpredictable frequency and may shorten the product life cycle. Beg and Chakraborty [11] observe another real life situation where the assumption about the finite planning horizon is questionable, e.g., modeling of seasonal products. They point out that for these type of products the decision making horizon may change due to environmental phenomena and therefore it is better to estimate this horizon as a stochastic parameter. Moon and Yun [12] consider an inventory model where the planning horizon is a random variable following an exponential distribution. This idea is explored by many scholars (e.g., Moon and Lee [13], Yu et al. [14]) and it is natural within a situation where the memory of product attributes and past experience with the product is irrelevant.

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https://doi.org/10.1016/j.camwa.2018.05.030
0898-1221/

Apart from the negative consequences of technological shocks on company performance such as increased adjustment costs of innovations, researchers observe benefits for the company. Song et al. [15] and Tsai et al. [16] draw attention to the possibility of increasing competitiveness, through the introduction of new technologically innovative products. Moreover, Shea in [17] shows the long-run positive relationship between technology innovation (such as R&D or patents) and growth in total factor productivity.

Therefore, understanding the mechanisms and effects of technological shocks is key to making strategic decision and maintaining a competitive market position. To meet the challenges we make an attempt to create a tool which enables us to designate optimal marketing strategies for different environmental conditions: changing frequency of occurrence of technological shocks and different levels of consumer recommendations, which are becoming an important source of information in making purchasing decisions (see Minnema et al. [18]).

To achieve these goals we investigate a model of product goodwill with the assumption that the planning horizon may change due to unpredictable technological turbulence. The first contribution to modeling of goodwill is made by Nerlove and Arrow [5]. They introduce the optimal advertising problem for a monopolistic firm maximizing profits, where the consumer demand function depends on the price of product and goodwill stock which summarize the past and present impact of advertising on demand. This model has been explored and developed in many areas. The modification of the classical approach is summarized in the comprehensive literature review given by Huang et al. [19]. Recently, the seminal Nerlove–Arrow model is the basis for describing the dynamics of goodwill in a competitive market. The latest results are summarized in He et al. [20] and Jørgensen and Zaccour [21]. Moreover, a goodwill model belongs to the wide class of vintage capital models (see Barucci and Gozzi [22] and Feichtinger et al. [23]). Grosset and Viscolani [24], and Faggian and Grosset [25] extend the seminal Nerlove–Arrow model by considering a company operating on a segmented market. As a result, the dynamics of goodwill in these models is described by a hyperbolic PDE.[1]

In Górajski and Machowska [9] these models are extended by new factors that are relevant for the economic description of the goodwill dynamics. Firstly, the market segmentation dependent on the usage experience. Secondly, goodwill is created and reinforced by positive consumer recommendations. Finally, a company invests in communication with existing and new consumers, using not only advertisement but also other marketing activities such as direct promotion, loyalty programs.

In this paper, we modify the optimal goodwill model considered in [9] to include a random product life cycle. This modification enable us for the first time to analyze marketing strategies dependent on the changeable decision-making horizon. Therefore, we are in a position to investigate long-term marketing strategies as a solution to an infinite horizon optimal control problem of a distributed system.

The remainder of this paper is organized as follows. Section 2 presents the reformulation of the product goodwill model in the case where the planning horizon is a random variable. Then, there are presented several properties related to the new goal functional. Section 3 proposes the definition of optimal marketing strategies and corresponding optimal goodwill. Next, the necessary and sufficient optimality conditions are formulated, goodwill long-run stationary equilibrium and marginal effects of technological shock frequency are investigated. Section 4 contains a numerical algorithm for computing an optimal solution and marginal effects. Next, the numerical example is provided in order to obtain sensitive behavior of the model. The final Section 5 incorporates the concluding remarks.

2. Goodwill model with random life cycle

Let $G(t,a)$ denote the stock of goodwill in the market segment a and time t, more precisely, we assume that goodwill is equal to the number of consumers who have been using the product for $a \in [0,1]$ units of time and they continue buying the product at time $t \geq 0$ as an effect of marketing activities. Hereinafter the market segmentation variable, a, is the time of product usage by the consumers and is called the consumer usage experience. It is associated with customers decisions. Since the consumer's life expectancy is limited, we assume that her experience in using this product is also limited (cf. banking or telephone services as an example of goods which may be used for almost a lifetime). However, in many cases, the consumer decides earlier to stop buying this product because it is no longer needed. Examples of such products may be baby diapers or medications that one reaches only when she is sick. Therefore, the consumer is able to collect only limited experience resulting from the time of using the product. We normalize the maximal usage experience to the value 1. Based on [9] we assume that goodwill satisfies the following partial differential equation of Lotka–Sharp–McKendrick type[2]

$$\begin{cases} \dfrac{\partial G(t,a)}{\partial t} = -\dfrac{\partial G(t,a)}{\partial a} - \delta(a)G(t,a) + \lambda^{\rho}(a)u^{\rho}(t,a), \\ G(t,0) = \displaystyle\int_0^1 (R(a)G(t,a) + R(a)\lambda^{\rho}(a)u^{\rho}(t,a))\,da + \lambda^{\rho}(0)u_0^{\rho}(t) \\ G(0,a) = G_0(a), \end{cases} \tag{1}$$

[1] Since the solution of partial differential equations (PDEs) reflects the distribution in space of a quantity, these systems are often also called distributed parameter systems (DPS) or infinite dimensional systems. For literature review on optimal control of distributed parameter systems we refer to Lions [26], Lasiecka and Triggiani [27], Bensoussan et al. [28] and reference therein.

[2] Known also as the von Foerster equation.

Please cite this article in press as: M. Górajski, D. Machowska, The effects of technological shocks in an optimal goodwill model with a random product life cycle, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.05.030.