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Moffatt eddies in the driven cavity: A quantification study by an HOC approach

Sougata Biswas, Jiten C. Kalita *

Department of Mathematics, Indian Institute of Technology Guwahati, PIN 781039, India

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ABSTRACT

In this paper, we establish Moffatt-likeness of the corner vortices in the lid-driven square cavity for Stokes flow and flow for moderate Reynolds numbers in the pre-asymptotic regime. The flow is computed by using an efficient transient Navier–Stokes (N–S) solver on compact non-uniform space grids. The quantification of the corner vortices in succession as Moffatt vortices follows from them following a fixed geometric ratio in sizes and intensities. In the process, we also provide more detailed benchmarking results for the corner vortices in the cavity flow. The accuracy of the scale resolution of the vortices has been verified by a novel approach to grid independence analysis. This approach utilizes the concept of adverse pressure gradients as a tool to validate that the separation zones in the neighborhood of the corners are consistent with the vortices obtained from our computed solution of the N–S equations.

1. Introduction

In the last few decades, the physical problem of the formation and evolution of vortices in fluids has been of great scientific interest for applied mathematicians, fluid dynamicists and physicists alike. More so, the corner vortices in the lid driven cavity flow which is probably the most celebrated problem [1–4] in the field of computational fluid dynamics. The popularity of this problem stems from the fact that it displays almost all fluid mechanical phenomena for incompressible viscous flows in the simplest of geometric settings. It has been primarily used as a benchmark problem for validating simulations resulting from the numerical solutions of the Navier–Stokes equations that govern the physics in incompressible viscous flows. Because of the frequency of use and enormous number of papers being published on this topic, it has almost reached the status of an *overstudied* problem. However, many facets of this problem still remain to be addressed by the scientific community. In the current study, we explore the possibility of categorizing the corner vortices for this flow as the so-called Moffatt vortices within numerical framework.

In fluid flow, vortices are considered as the most important structures as it controls the dynamics of the flow [5]. They are known to form and occur in the vicinity of solid walls where the viscous effect is felt largely. The existence of a sequence of vortices at the corner of solid structure for internal flows with decreasing size and rapidly decreasing intensity towards the corner has been indicated by physical experiments and mathematical asymptotics. Such vortices for Stokes flow between two solid boundaries were first established theoretically by H. K. Moffatt [6,7] and are fittingly referred to as "Moffatt vortices" in the existing literature. Following this, the existence of Moffatt vortices in slow viscous incompressible flows on different geometries [6–18] has so far been established mostly through theoretical studies. There have been very few experimental and

* Corresponding author.
E-mail addresses: b.sougata@iitg.ernet.in (S. Biswas), jiten@iitg.ernet.in (J.C. Kalita).

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numerical studies [9,10,19,20] on this topic. Most theoretical studies on Moffatt vortices seek the solution of the biharmonic form of the N–S equations for Stokes flow [21]

$$\nabla^4 \psi = 0 \tag{1}$$

which is a linear one. The solution is assumed to be of the form $\psi = \tilde{r}^{\Lambda} f(\phi)$ leading to an equation in f [21]

$$f'''' + \left\{(\Lambda+1)^2 + (\Lambda-1)^2\right\} f'' + (\Lambda+1)^2(\Lambda-1)^2 f = 0, \tag{2}$$

resulting in a solution of the form [6,7,21,22]

$$f(\phi) = C_1 \sin(\Lambda-1)\phi + C_2 \cos(\Lambda-1)\phi + C_3 \sin(\Lambda+1)\phi + C_4 \cos(\Lambda+1)\phi, \tag{3}$$

where Λ is some exponential power of the distance $\tilde{r}$ from the corner. This Λ was found to be a complex number when the angle between the two planes does not cross a certain limit, implying infinite oscillations, i.e. an infinite sequence of counter-rotating eddies as the corner is approached. The sizes and intensities of two successive vortices in this sequence asymptotically approach a fixed ratio.

The numerical computation of incompressible viscous flows essentially consists of solving the Navier–Stokes (N–S) equations which provide a framework for exploring different facets of the vortex dynamics. As mentioned earlier, the study of Moffatt vortices has been confined mainly to Stokes flow. The mention of Moffatt vortices in the numerical computation of incompressible fluid flow for moderate Reynolds number dates back to the work of Burgraff [23] in 1966, who besides computing the lid-driven square cavity flow, provided a linearized analysis of circular eddies. In 1967, Pan and Acrivos [24] computed flow in rectangular cavities and made use of interpolation techniques to locate the strength and location of tertiary vortices and eventually calculate the common ratio of the vortex sizes. In their work, Dennis and Collins [10] used a second order central difference formula in finite difference set up for numerically computing the corner vortices in the flow through a curved tube of triangular cross section. They endeavored to achieve the theoretical limits of the intensity and size ratio of the vortices through their numerical studies. Nonetheless, instead of computing the entire flow in the physical domain, they were extrapolating data from the bigger vortices to compute the smaller ones in succession as done by Pan and Acrivos [24]. In 2004, Biswas et al. [9] employed the SIMPLE algorithm [25] on a finite volume setup for computing flow in the backward facing step problem. However, their quantification of Moffatt vortices at the corner was confined to the mention of finding multiple vortices at the backward step corner and the possibility of its existence as $Re \rightarrow 0$ only. Recently, Magalhães et al. [26] compared the Stokes flow parameters found by Moffatt [6,7] with the data available from last possible corner vortices for a single Reynolds number value 1000 through an adaptive mesh finite volume computation. Likewise, Biswas and Kalita [27] initiated a study to quantify the corner vortices for the same flow as Moffatt; however it was confined to $Re = 100$ only.

The objective of the current study is to rigorously explore the prospect of quantifying the corner vortices in the famous lid-driven square cavity problem as Moffatt: whether it exists only for Stokes flow or flows at moderate and high Reynolds numbers also. The essential problem with the characterization of vortices as Moffatt eddies lies in finding the asymptotic relation between successive vortices. However, in the numerical framework, no matter how strong the computational resources are, it is impossible to obtain data for the extreme end of the sequence of vortices. As the vortices of greatest interest are the weakest ones belonging to this extreme end, it is highly unlikely that numerical studies alone will be sufficient to answer these questions. As such, the current study is more an endeavor towards establishing Moffatt-likeness of the sequence of vortices in the pre-asymptotic regime. Within the limitations of the computational resources available at our disposal, we would explore the possibility of characterizing these vortices as Moffatt based on the first few corner vortices that has been possible to capture through the current computation.

It is well known that in order to capture the vortices in the extreme corners belonging to the sequence of Moffatt vortices, one not only needs a strong numerical scheme, but also the finest grids possible in the neighborhood of the corners. In order to accomplish this, we utilize the recently developed Higher Order Compact (HOC) scheme by Kalita et al. [28] on non-uniform grids with extreme clustering. Computation of incompressible viscous flows by HOC schemes [28–32] has gained momentum over the past few years owing to their computational efficiency. Though these schemes have been widely used for the numerical simulation of a host of fluid flow problems, to the best of our knowledge, no HOC scheme has yet been utilized for analyzing the existence of Moffatt vortices, more so for the lid-driven cavity flow. The simulation prompted us to explore a broad categorization of the corner vortices, whose dimensions and intensities in succession should ideally fall off in geometric progression with fixed geometric ratios for them to qualify as Moffatt. We further introduce a novel approach to grid independence studies in order to confirm that the smallest scales captured by us are not numerical artifacts.

The paper is arranged in the following manner: Section 2 deals with the problem description, Section 3 with the basic formulation and numerical procedures, Section 4 with grid independence and benchmarking, Section 5 with the size and intensity ratios of corner vortices and finally Section 6, the conclusions.