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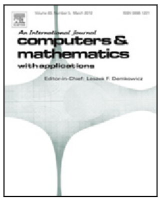

Several embedded cracks in a functionally graded piezoelectric strip under dynamic loading

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ABSTRACT

This paper deals with the fracture analysis of multiple cracks in a functionally graded piezoelectric (FGP) layer under applied anti-plane time-harmonic mechanical and in-plane electrical loading within the framework of linear electro-elasticity. The analysis is based on the stress and electric fields caused by Volterra-type screw dislocation in the medium. The crack surfaces are assumed to be impermeable condition. The material properties of the medium are considered to obey exponential variations. In order to model the cracked layer, the distributed dislocation technique is employed to perform Cauchy-type singular integral equations for layer, in which the unknown variables are dislocation densities. The dislocation densities are then employed to derive field intensity factors at the crack tips. The results show that the stress and the electric displacement intensity factors at the crack tips depend on the cracks configuration, frequency and material properties. Finally, several numerical examples are solved to obtain the dynamic stress intensity factors (DSIFs) and electric displacement intensity factors.

1. Introduction

In the past two decades, the passive elastic and active properties of piezoelectric materials and shape memory alloys have drawn a great deal of attention and have been employed widely in structural enhancement [1,2]. The electro-mechanical properties of functionally graded piezoelectric (FGP) materials vary continuously in certain directions to compensate sharp changes in electro mechanical fields which indeed boils down to catastrophic failures. FGP materials make them an eye-catching choice in many high-tech applications due to their promising properties. For instance, they are widely used in electromechanical and electronic devices such as spacecraft, electromechanical transducers, electronic packaging, solar projector, thermal sensors, medical ultrasonic imaging, active noise and vibration suppression of aircraft wings, position control of flexible robot arms, smart skin systems for submarines and shape control of advanced structures. It, however, is widely known that the piezoelectric ceramics are very brittle with low toughness. As a result, because of their behavior, these materials are far more vulnerable to crack. There has been a plethora of research regarding these materials in the field of fracture mechanics. It, also, should be noted that because of their diverse applications, piezoelectric materials have been studied under various types of loadings namely; static [3], time-harmonic [4–6] and transient [7] loadings types. Fracture analysis of piezoelectric materials usually contains two kinds of boundary conditions known as the electrically impermeable and permeable on crack faces [8]. As a matter of fact, electromechanical loadings is the reason that the crack opens, while the electric field in the crack cavity significantly depends on the crack opening and the permeability of the material in the crack cavity. Consequently, the crack opening model [9] considering the electric field in the crack cavity, which is sometimes called

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Nomenclature

$A_1(s), A_2(s), A_3(s), A_4(s), C_1(s), C_2(s), C_3(s), C_4(s)$	Unknown coefficients
b_{mz}, b_{pz}	Burgers vector
B_{mzj}, B_{pzj}	Dislocation densities
c	Transverse shear wave velocity
$c_{44}(y)$	Elastic modulus
D_x, D_y	Electric displacements
$g_{mzj}(q), g_{pzj}(q)$	Regular terms of dislocation densities
h	Thickness of strip
$H(x)$	Heaviside step function
$(K_{III}^{D})_{Li}$	Electric intensity factor of left side of crack
$(K_{III}^{D})_{Ri}$	Electric intensity factor of right side of crack
$(K_{III}^{m})_{Li}$	Stress intensity factor of left side of crack
$(K_{III}^{m})_{Ri}$	Stress intensity factor of right side of crack
K_{0D}	Electric intensity factor of a crack in infinite plane
K_{0M}	Stress intensity factor of a crack in infinite plane
$K_{ij}(s, t)$	Kernels of integral equations
l	Half lengths of straight crack
N	Total number of cracks
r_{Li}, r_{Ri}	Distance from left and right crack tips
s	Fourier variable
W, w	Out of plane displacement component
$x_i(p), y_i(p)$	Functions describing the geometry of cracks
$\delta(s)$	Dirac delta function
λ	FGM exponent
ρ	Mass density
ϕ	Electric potential
σ_{nz}	Traction vector
σ_{zx}, σ_{zy}	Out of plane stress components
(η, ξ)	Coordinates describing the dislocation line
ω	Angular frequency

the semi-permeable crack [8], is much more suitable to the facts. Diverse electro-elastic behaviors of a dielectric crack have been investigated by many researchers who took the effect of dielectric permittivity into account [10–16]. The problem of a finite crack in a strip of FGPM was studied by Li and Weng [17]. The elastic stiffness, piezoelectric constant, and dielectric permittivity of the FGPM are assumed to vary continuously along the thickness of the strip which is under an anti-plane mechanical loading and in-plane electric loading. Wang [18] considered the mode III crack problem in FGPM medium. The mechanical and the electrical properties of the medium are considered in a way that the equilibrium equations have an analytical solution. In this investigation, the influence of material inhomogeneity parameter on SIFs for both single crack and series of collinear cracks is studied.

The problem for a finite crack propagating at constant speed in a FGP ceramic strip under anti-plane shear mechanical and in-plane electrical loads is studied by Kwon [19]. The influences of the crack propagation speed, electric field, FGP material parameter, crack length, and electromechanical coupling coefficient were studied on SIFs. The anti-plane problem of functionally graded piezoelectric strip with a constant-velocity Yoffe-type moving crack under the permeable and impermeable electric condition is investigated by Hu and Zhong [20]. The effects of the material properties and the crack velocity were studied on the stress distribution near the crack tip. The in-plane crack problem of FGP solids under time-harmonic loading was investigated by Dineva et al. [21]. The material parameters are assumed to vary quadratically with both spatial variables. The effects of the inhomogeneity parameters, the frequency of the applied electromechanical load and the geometry of the crack on SIFs were investigated. Guo et al. [22] studied the fracture behavior of multiple cracks emerging from a circular hole in piezoelectric material under in-plane electric and anti-plane mechanical loadings. The problem of determining the electro-elastic field around arbitrarily oriented planar cracks in an infinite piezoelectric space is considered by Ang and Athanasius [23]. The cracks under transient loadings are considered to be electrically impermeable or permeable. Asadi [24] defined an axisymmetric annular electric dislocation and studied the field intensity factors for a system of interacting annular and/or penny-shaped cracks. The axisymmetric electric dislocation together with Volterra climb and glide edge dislocations in an infinite transversely isotropic piezoelectric medium are determined. Permeable and impermeable electric boundary conditions on the crack faces under axisymmetric electromechanical loading were investigated. The problem of a cracked FGP strip under anti-plane mechanical and in-plane electrical loading is studied by

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