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Rogue waves for a variable-coefficient Kadomtsev–Petviashvili equation in fluid mechanics

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ABSTRACT

Under investigation in this paper is a variable-coefficient Kadomtsev–Petviashvili equation in fluid mechanics, which describes the shallow-water waves with the weak nonlinearity and dispersion. Employing the Kadomtsev–Petviashvili hierarchy reduction, we obtain the rogue-wave solutions in terms of the Gramian. Periodic, cubic- and s-shaped line rogue waves are presented with different forms of the dispersion coefficient. The second-order rogue waves and multi-rogue waves are also graphically discussed. It is observed that only parts of the second-order rogue wave approach the constant background, and the other parts move to the far distance with the undiminished amplitudes. The multi-rogue waves describe the interaction of several first-order rogue waves. We plot the interactions of two periodic, cubic- and s-shaped line rogue waves. The lump wave, which propagates stably in all directions, is also depicted.

1. Introduction

As a special type of nonlinear waves, rogue waves, which have been first reported as the extreme event emerging unexpectedly from the ocean with destructive power, have triggered the interest in recent years [1,2]. Different from the solitons and breathers [3–5], a rogue wave is the isolated wave with its amplitude two to three times higher than its surrounding waves and forms in a short time [1,2]. Rogue waves have been experimental observed in the plasmas [6], Bose–Einstein condensates [7] and optical fibers [8,9]. Higher-order rogue waves have been generated in a water-wave experiment [10].

Rogue waves have been described via the rational solutions for the nonlinear evolution equations (NLEEs) [11–14]. Rogue waves for the Davey–Stewartson equations [15,16], Sasa–Satsuma equation [17], discrete and continuous nonlinear Schrödinger equations [18,19] have been analytically studied. In addition, researchers have noticed that the variable-coefficient NLEEs can describe the real situations more precisely than the constant-coefficient ones [20,21]. For example, people have considered the following variable-coefficient Kadomtsev–Petviashvili (KP) equation for the shallow-water waves with the weak nonlinearity and dispersion [22–25]:

$$[u_t + l(t)uu_x + g(t)u_{xxx} + \mu(t)u]_x + h(t)u_{yy} = 0, \qquad (1)$$

where u, a differentiable function of the scaled space coordinates x, y and time coordinate t, represents the wave amplitude, the real function $\mu(t)$ gives the perturbed effect, the real functions $l(t)$, $g(t)$ and $h(t)$ represent the coefficients of nonlinearity,

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dispersion and disturbed term along the y direction, respectively. Under the integrable constraints

$$g(t) = Cl(t)e^{-\int \mu(t)dt}, \quad h(t) = \lambda g(t), \tag{2}$$

and through the transformation

$$u = 12Ce^{-\int \mu(t)dt}(\ln f)_{xx}, \tag{3}$$

bilinear form for Eq. (1) has been obtained as [23–25]

$$\left[D_x^4 + \frac{1}{g(t)}D_xD_t + \lambda D_y^2\right] f \cdot f = 0, \tag{4}$$

where C, λ (< 0) are both the real constants, $f(x, y, t)$ is a real positive function, D_x, D_y and D_t are the bilinear differential operators [26], defined by

$$D_x^l D_y^m D_t^o (F \cdot G) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^o F(x, y, t)G(x', y', t')|_{x'=x, y'=y, t'=t},$$

with x', y' and t' as the formal variables, $F(x, y, t)$ and $G(x', y', t')$ being two functions, l, m and o being the non-negative integers. Particularly, $f(x, y, t) = \phi(y, t)e^{at}$, where $\phi(y, t)$ is a function to be determined and a is a constant, implies a trivial solution for Bilinear Form (4). Based on Bilinear Form (4), multi-soliton solutions, Lax pairs and Bäcklund transformations for Eq. (1) under Integrable Constraints (2) have been presented [22–24]. N-soliton solutions, Wronskian and Gramian solutions for Eq. (1) have been derived [23,25]. More on the variable-coefficient KP issues can be seen, e.g., in Refs. [27].

However, to our knowledge, multi- and higher-order rogue waves for Eq. (1) have not been investigated before. In this paper, by virtue of the KP hierarchy reduction [15,26], we will derive the rational solutions in terms of the Gramian in Section 2. Based on such rational solutions, rogue waves will be graphically analyzed in Section 3. Section 4 will provide our conclusions.

2. Rational solutions for Eq. (1)

In order to obtain the rogue waves for Eq. (1), we will present the solutions for Bilinear Form (4) in the following theorem.

Theorem 1. *Solutions in terms of the Gramian for Bilinear Forms (4) can be given by the $N \times N$ determinants,*

$$f(x, y, t) = \det_{1 \le i,j \le N} \left(m_{ij}\right), \tag{5}$$

$$m_{ij} = \left\{ \sum_{k=0}^{n_i} c_{ik}\left[p_i \partial_{p_i} + \xi_i'(x, y, t)\right]^{n_i - k} \sum_{l=0}^{n_j} c_{jl}^*\left[q_j \partial_{q_j} + \eta_j'(x, y, t)\right]^{n_j - l} \right\} \frac{1}{p_i + q_j}\Bigg|_{p_i = p_i', q_i = p_i'^*},$$

where ξ_i''s and η_j''s are the functions with respect to variables x, y and t, defined as

$$\xi_i'(x, y, t) = p_i x + 2\sqrt{3/\lambda}p_i^2 y - 12p_i^3 \int g(t)dt,$$

$$\eta_j'(x, y, t) = q_j x - 2\sqrt{3/\lambda}q_j^2 y - 12q_j^3 \int g(t)dt,$$

i, j and n_i's are the integers, p_i's and q_i's are the complex variables, c_{ij}'s and p_i''s are the complex constants, and "$*$" means the complex conjugate. The proof of Theorem 1 is given in Appendix A.

By setting $N = 1$ and $n_1 = 1$, we rewrite Solutions (5) as follows:

$$f(x, y, t) = \left[p_1 \partial_{p_1} + \xi_1'(x, y, t) + c_{11}\right]\left[q_1 \partial_{q_1} + \eta_1'(x, y, t) + c_{11}^*\right] \frac{1}{p_1 + q_1}\Bigg|_{p_1 = p_1', q_1 = p_1'^*} \tag{6}$$

$$= \frac{1}{p_1' + p_1'^*}\left[\left|\tilde{\xi_1}(x, y, t) + c_{11} - \frac{p_1'}{p_1' + p_1'^*}\right|^2 + \frac{p_1' p_1'^*}{(p_1' + p_1'^*)^2}\right],$$

with

$$\tilde{\xi_1}(x, y, t) = p_1' x + 2\sqrt{3/\lambda}p_1'^2 y - 12p_1'^3 \int g(t)dt.$$