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Anomalous diffusion in comb model with fractional dual-phase-lag constitutive relation

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ABSTRACT

A fractional dual-phase-lag constitutive relation is proposed to describe the anomalous diffusion in comb model. A novel governing equation with the Dirac delta function is formulated and the highest order is $1+\alpha$ which corresponds to a diffusion-wave equation. Solutions are obtained analytically with Laplace and Fourier transforms. Dynamic characteristics for the spatial and temporal evolution of particle distribution and the mean square displacement versus time with the effects of different parameters such as the fractional parameters and the relaxation parameters are analyzed and discussed in detail. Results show that the wave characteristic becomes stronger for a larger fractional parameter, a smaller microscopic relaxation parameter or a larger macroscopic one. For a larger α, a smaller β, a larger macroscopic relaxation parameter or a smaller microscopic one, a novel oscillating distribution versus time is presented, and at this condition, the magnitude of mean square displacement is larger at the smaller time while larger at the larger time. Most important of all, the anomalous diffusion in comb model with a diffusion-wave equation corresponds to a subdiffusion behavior because of its special structure.

1. Introduction

As a classical model to study the anomalous diffusion, the comb model has widespread research backgrounds. It is originally proposed to mimic percolation clusters [1,2]. With the development of science and technology, it is further applied on the diffusion of cancer cells [3,4], the transport along spiny dendrites [5,6], the reaction front propagation of actin polymerization [7], and so on. As Fig. 1 shows, the comb model consists of a one-dimensional backbone and lateral branches [8]. Due to its geometrical construction, a special behavior [9] is that the transport along the x direction only happens along the x axis with the diffusion coefficient $D_1\delta(y)$, while the diffusion in the y direction plays a role of traps with the traditional diffusion coefficient D_0, here, $\delta(y)$ refers to the Dirac delta function, D_1 and D_0 are constants.

A random walk on the comb model is described by the distribution function $P(x, y, t)$ and the current [3,10]:

$$\vec{J} = \left(-D_1\delta(y)\frac{\partial P}{\partial x}, -D_0\frac{\partial P}{\partial y}\right). \tag{1}$$

Since the sizes and shapes of fingers and backbone are completely random [11], and the fingers act as a barrier [12], the transport is assumed anomalous that the classical model cannot satisfy to describe the diffusion in comb model. Besides, the classical Fick's model contains paradox which corresponds to the infinite propagation velocity [13]. To find

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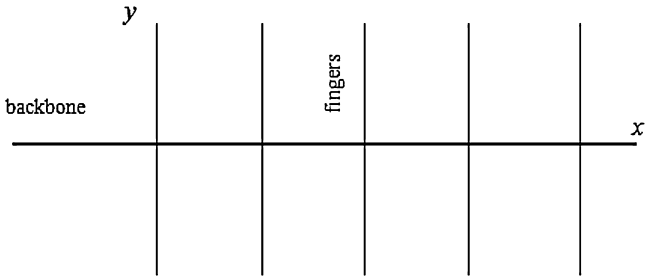

Fig. 1. Schematic drawing of the comb model.

a proper constitutive relationship is the basis for the study of anomalous diffusion in comb model. Many attempts have been made to modify the traditional model. Introducing the macroscopic relaxation parameter, Cattaneo [14] proposed a new hyperbolic model with finite propagation velocity, covering a wide range of practical problems, such as the hyperbolic diffusion problems with the pulsed boundary conditions [15], the transient fractional heat conduction [16], and the unsteady Marangoni convection heat transfer of fractional Maxwell fluid [17].

However, the Cattaneo model only considers the fast-transient effects among the macrostructure, but not considers the interactions in microstructure [18]. In order to give the macroscopic description with microscopic effects, Tzou [19,20] proposed the dual-phase-lag model which bridged the gap between the macroscopic approach and the microscopic approach. The modified model has been widely used to describe the practical problems and becomes a rapidly growing area for its applications in a wide range of fields of science and engineering. Based on the experiment of heat propagation in processed meat [21], Antaki [22] offered a new interpretation for experimental evidence with dual phase lag model that was interpreted previously with hyperbolic conduction. As an extension study of Antaki [22], Liu and Chen [23] applied the dual phase lag model of heat conduction to interpret the non-Fourier thermal behavior. Zhang [24] obtained the dual-phase-lag bioheat equations with blood or tissue temperature as sole unknown temperature, finding that the phase lag times for heat flux were very close to the temperature gradient for the living tissue.

The studies mentioned above mainly analyze the practical problems with local integer constitution model which is dependent on its nearby points. As a development of the integer one, the fractional derivative is a nonlocal one which considers memory and nonlocal characteristics [25]. It has widespread applications and has attracted many scholars' attention. Among the researchers, Song and Jiang [26] studied the viscoelastic fluids with fractional Jeffreys model, results showed that the measured dynamic moduli and the theoretical predictions agreed very well. Based on the realistic property of the pore structure in coal, Jiang et al. [27] indicated that the fractional diffusion model fitted better than the Fick's classical model by three measured samples. Chen et al. [28] described the coupled chloride diffusion-binding processes in reinforced concrete with a new variable-order fractional diffusion equation model, illustrating that the proposed fractional model agreed significantly better with experimental data through four test cases. Xu and Chen [29] studied the complex viscoelastic creep behaviors of Hami Melon by a fractional derivative model which was more efficient and accurate than the generalized Kelvin viscoelastic model.

The fractional constitutive models, such as the fractional Fick's model [30], the fractional Cattaneo model [31] and the fractional Cattaneo–Christov model [32] have been proposed to study the anomalous diffusion in comb model. For the fractional dual-phase-lag model, it has been applied in the heat transfer within skin tissue [33,34]. However, for the mass diffusion, the two-dimensional fractional dual-phase-lag model containing both the relaxation parameters and the fractional parameters has never been involved.

2. Mathematical formulation

Motivated by the above discussions, the fractional dual-phase-lag model is firstly proposed to study the anomalous diffusion in comb model. By referring to [33,34], the fractional dual-phase-lag constitutive equation to describe the anomalous diffusion in comb model is given as:

$$\overrightarrow{J} + \tau_q \frac{\partial^\alpha \overrightarrow{J}}{\partial t^\alpha} = \left(1 + \tau_P \frac{\partial^\beta}{\partial t^\beta}\right)\left(-D_1 \delta(y) \frac{\partial P}{\partial x}, -D_0 \frac{\partial P}{\partial y}\right), \tag{2}$$

where τ_q is the macroscopic relaxation parameter, τ_P refers to the microscopic relaxation parameter, $\frac{\partial^\alpha}{\partial t^\alpha}$ and $\frac{\partial^\beta}{\partial t^\beta}$ are the Caputo fractional derivatives of order α $(0 < \alpha \le 1)$ and β $(0 < \beta \le 1)$, respectively. The definition [35,36] is given as:

$$\frac{\partial^{\alpha_1} P(x, y, t)}{\partial t^{\alpha_1}} = \frac{1}{\Gamma(1-\alpha_1)} \int_0^t \frac{1}{(t-\tau)^{\alpha_1}} \frac{\partial P(x, y, \tau)}{\partial \tau} d\tau, \tag{3}$$