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Global nonexistence of solutions for a nonlinear wave equation with time delay and acoustic boundary conditions

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ABSTRACT

We consider a quasilinear wave equation

$$\begin{aligned} u_{tt} - \triangle u_t - div(|\nabla u|^{\alpha-2}\nabla u) - div(|\nabla u_t|^{\beta-2}\nabla u_t) \\ +a|u_t|^{m-2}u_t + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = b|u|^{p-2}u \end{aligned}$$

$a, b > 0$, associated with initial and Dirichlet boundary conditions at one part and acoustic boundary conditions at another part, respectively. We prove, under suitable conditions on α, β, m, p and for negative initial energy, a global nonexistence of solutions.

1. Introduction

In this paper, we consider with following nonlinear wave equation with time delay and acoustic boundary conditions:

$$\begin{aligned} &u_{tt} - \triangle u_t - div(|\nabla u|^{\alpha-2}\nabla u) - div(|\nabla u_t|^{\beta-2}\nabla u_t) \\ &\quad + a|u_t|^{m-2}u_t + \mu_1 u_t(x,t) + \mu_2 u_t(x,t-\tau) = b|u|^{p-2}u, \text{ in } \Omega \times (0,\infty), \end{aligned} \tag{1.1}$$

$$u = 0, \text{ on } \Gamma_0 \times (0,\infty), \tag{1.2}$$

$$\frac{\partial u_t}{\partial \nu} + |\nabla u|^{\alpha-2}\frac{\partial u}{\partial \nu} + |\nabla u_t|^{\beta-2}\frac{\partial u_t}{\partial \nu} = h(x)y_t, \text{ on } \Gamma_1 \times (0,\infty), \tag{1.3}$$

$$u_t + f(x)y_t + q(x)y = 0 \text{ on } \Gamma_1 \times (0,\infty), \tag{1.4}$$

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x) \text{ in } \Omega, \tag{1.5}$$

$$u_t(x,t-\tau) = f_0(x,t-\tau) \text{ in } \Omega \times (0,\tau), \tag{1.6}$$

$$y(x,0) = y_0(x), \text{ on } \Gamma_1, \tag{1.7}$$

where $a, b > 0, \alpha, \beta, m, p > 2, \mu_1 > 0, \mu_2$ is real number, and $\tau > 0$ represents the time delay. Ω is a regular and bounded domain of $R^n (n \geq 1)$ and $\partial\Omega (:= \Gamma) = \Gamma_0 \cup \Gamma_1$. Here Γ_0, Γ_1 are closed and disjoint and $\frac{\partial}{\partial \nu}$ denotes the unit outer normal derivative. The functions $f, q, h : \Gamma_1 \longrightarrow R_+ (:= [0,\infty))$ are essentially bounded and $0 < q_0 \leq q(x)$ on Γ_1. Time delay arises in many applications because, in most instances, physical, chemical, biological, thermal, and economic phenomena naturally

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not only depend on the present state but also on some past occurrences. In particular, the effects of time delay strikes on our system have a significant effect on the range of existence and the stability of the system. And this is especially important for feedback control problem. In recent years, the control of partial differential equations with time delay effects has become an active area of research(see [1–3]). In [4], Kirane and Said-Houari consider the following linear viscoelastic wave equation with a linear damping and a delay term

$$\begin{aligned}&u_{tt}(x,t)-\triangle u(x,t)+\int_0^t g(t-s)\Delta u(x,s)ds\\&\quad+\mu_1 u_t(x,t)+\mu_2 u_t(x,t-\tau)=0,\ \text{in }\Omega\times(0,\infty),\\&u(x,t)=0,\ \text{on }\partial\Omega\times(0,\infty),\\&u(x,0)=u_0(x),u_t(x,0)=u_1(x)\text{ in }\Omega,\\&u_t(x,t-\tau)=f_0(x,t-\tau)\text{ in }\Omega\times(0,\tau),\end{aligned}$$

where Ω is a regular and bounded domain $R^n(n\geq 1)$, μ_1,μ_2 are positive constants, $\tau>0$ represents the time delay and u_0,u_1,f_0 are given functions belonging to suitable space. They proved the existence and asymptotic stability of a viscoelastic wave equation with delay. And Park et al. [5,6] deal with the existence, energy decay and stability for the equation of memory type with delay. Furthermore, in other type, Jleli et al. [7,8] showed the nonexistence of global solutions for the nonlinear evolution equation/the Schrodinger equation. Kafini and Messaudi [9] studied the following nonlinear damping wave equation with delay

$$\begin{aligned}&u_{tt}(x,t)-div(|\nabla u(x,t)|^{m-2}\nabla u(x,t))+\mu_1 u_t(x,t)+\mu_2 u_t(x,t-\tau)=b|u|^{p-2}u,\ \text{in }\Omega\times(0,\infty),\\&u(x,t)=0,\ \text{in }\partial\Omega\times(0,\infty),\\&u(x,0)=u_0(x),u_t(x,0)=u_1(x)\text{ in }\Omega,\\&u_t(x,t-\tau)=f_0(x,t-\tau)\text{ in }\Omega\times(0,\tau),\end{aligned}$$

where $p>m\geq 2$, b,μ_1 are positive constants, μ_2 is a real number, and $\tau>0$ represents the time delay. The authors established the blow up result in a nonlinear wave equation with delay. We also consider the acoustic boundary conditions. The physical applications of the acoustic boundary conditions are related to the problem of noise control and applications (see [10–19]). The acoustic boundary conditions were introduced by Morse and Ingard [20] in 1968 and developed by Beale and Rosencrans in [21], where the authors proved the global existence and regularity of the linear problem. Graber and Said-Houari [22], consider the model of wave equation with semilinear porous acoustic boundary conditions with nonlinear damping

$$\begin{aligned}&u_{tt}(x,t)-\triangle u(x,t)+\alpha(x)u(x,t)+\Phi(u_t(x,t))=j_1(u(x,t))\text{ in }\Omega\times(0,\infty),\\&u=0,\ \text{on }\Gamma_0\times(0,\infty),\\&\frac{\partial u(x,t)}{\partial\nu}-h(x)\eta(z_t(x,t))+\rho(u_t(x,t))=j_2(u(x,t)),\ \text{on }\Gamma_1\times(0,\infty),\\&u_t(x,t)+f(x)z_t(x,t)+g(x)y(x,t)=0\text{ on }\Gamma_1\times(0,\infty),\\&u(x,0)=u_0(x),u_t(x,0)=u_1(x)\text{ in }\Omega,\\&z(x,0)=z_0(x),\ \text{on }\Gamma_1,\end{aligned}$$

where Ω is a regular and bounded domain $R^n(n\geq 1)$ with boundary Γ of class C^2, $\alpha:\Omega\to R$ and $f,g,h,\rho,j_2:\overline{\Gamma}_1\to R$ and also $\Phi,j_1:\Omega\to R$ are given functions. The authors studied the existence, energy decay and blow up result of solutions for wave equation with semilinear porous acoustic boundary conditions. For more examples, see [1–3]. Recently, Jeong et al. [23] proved the nonexistence of solutions for a nonlinear wave equation with acoustic boundary conditions

$$\begin{aligned}&u_{tt}(x,t)-\triangle u_t(x,t)-div(|\nabla u(x,t)|^{\alpha-2}\nabla u(x,t))-div(|\nabla u_t(x,t)|^{\beta-2}\nabla u_t(x,t))\\&\quad+a|u_t(x,t)|^{m-2}u_t(x,t)=b|u(x,t)|^{p-2}u(x,t),\ \text{in }\Omega\times(0,\infty),\\&u(x,t)=0,\ \text{on }\Gamma_0\times(0,\infty),\\&\frac{\partial u_t(x,t)}{\partial\nu}+|\nabla u(x,t)|^{\alpha-2}\frac{\partial u(x,t)}{\partial\nu}+|\nabla u_t(x,t)|^{\beta-2}\frac{\partial u_t(x,t)}{\partial\nu}=h(x)y_t(x,t),\ \text{on }\Gamma_1\times(0,\infty),\\&u_t(x,t)+f(x)y_t(x,t)+q(x)y(x,t)=0\text{ on }\Gamma_1\times(0,\infty),\\&u(x,0)=u_0(x),u_t(x,0)=u_1(x)\text{ in }\Omega,\\&y(x,0)=y_0(x),\ \text{on }\Gamma_1,\end{aligned}$$

where $a,b>0,\alpha,\beta,m,p>2$, Ω is a regular and bounded domain of $R^n(n\geq 1)$ and $\partial\Omega(:=\Gamma)=\Gamma_0\cup\Gamma_1$. Here Γ_0,Γ_1 are closed and disjoint, and $\frac{\partial}{\partial\nu}$ denotes the unit outer normal derivative. The functions $f,q,h:\Gamma_1\longrightarrow R_+$ are essentially bounded and $0<q_0\leq q(x)$ on Γ_1.

The paper studied above became a motivation, and in this paper, we investigate a blow up of solution for a nonlinear wave equation with time delay and acoustic boundary conditions. To the best of our knowledge, this result is the first work that deals with blow up solutions to problem involving time delay and acoustic boundary conditions. Thus the result is meaningful. The contents of this paper is as follows. In Section 2, we prepare some material needed in our proof and state the energy functional. In Section 3, we prove our main result.

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