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Performance assessment, diagnosis, and optimal selection of non-linear state filters☆

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ABSTRACT

Non-linear state filters of different approximations and capabilities allow for real-time estimation of unmeasured states in non-linear stochastic processes. It is well known that the performance of non-linear filters depends on the underlying numerical and statistical approximations used in their design. Despite the theoretical and practical interest in evaluating the performance of non-linear filtering methods, it remains one of the most complex problems in the area of state estimation. We propose the use of posterior Cramér–Rao lower bound (PCRLB) or mean square error (MSE) inequality as a filtering performance benchmark. Using the PCRLB inequality, we develop assessment and diagnosis tools for monitoring and evaluating the performance of non-linear filters. Using the PCRLB inequality-based performance assessment tool, an optimal non-linear filter switching strategy is proposed for state estimation in general non-linear systems. The non-linear filter switching strategy is an optimal performance strategy, which maintains high filtering performance under all operating conditions. The complex, high dimensional integrals involved in the computation of the PCRLB inequality-based non-linear filter assessment and diagnosis tools are approximated using sequential Monte-Carlo (SMC) methods. The utility and efficacy of the developed tools are illustrated through a numerical example.

1. Introduction

Recent advances in high-speed computing technology have enabled the process and manufacturing industries to use complex and high-fidelity non-linear dynamical models, such as in fermentation bioreactors [2], polymerization reactors [3], and petroleum reservoirs [4]. The implementation of advanced control and monitoring strategies on such complex systems requires measurement of the key process state variables, which in many processes are often hidden or unmeasured. These unmeasured states can be estimated within the Bayesian framework by solving a filtering problem; wherein, a posterior density for the states is recursively computed at each sampling-time, conditioned on the available measurement sequence [5]. In linear filtering, the state posterior density is Gaussian, and can be exactly represented by the Kalman filter (KF) using a finite number of moments (e.g., mean, variance); whereas, in non-linear filtering, where the posterior is non-Gaussian, at least in theory, an infinite number of moments are required for exact representation of the density [6]. Thus, with finite computing capabilities, an optimal non-linear state filter is not realizable.

In the last few decades, several approximate non-linear state filters[1] based on statistical and analytical approximations of the optimal non-linear filter have been developed for state estimation in stochastic non-linear state-space models (SSMs) [7,8]. Most of these non-linear filters can be classified as either Kalman-based filters (e.g., extended KF (EKF), unscented KF (UKF), ensemble KF (EnKF)) or SMC-based filters (e.g., sequential importance resampling (SIR) filter, auxiliary SIR (ASIR) filter, Rao–Blackwellized particle filter (RBPF)). Both the Kalman and SMC-based filters are tractable in finite computational time and can be used for state estimation in general or specific types of non-linear SSMs; however, their filtering performance (compared to the optimal non-linear filter) is often constrained by the underlying numerical and statistical approximations. A detailed exposition of non-linear filtering

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[1] For convenience, an approximate non-linear state filter will here onwards be referred to as a non-linear filter.

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methods and related approximations is not included here, but can be found in the handbook of non-linear filtering [9].

A recent surge of interest in developing advanced numerical methods for state estimation in non-linear SSMs has left researchers and practitioners inundated with a large number of non-linear filters to choose from. To allow researchers to develop numerically efficient filters, and practitioners to select the right filtering strategy for state estimation in their process, several authors – [10–12] – have considered comparing and analysing the filtering performance of different non-linear filters on general non-linear systems. Many application specific comparisons of non-linear filters have also been established for systems in process industries [13–15], aerospace and aeronautics [16,17], and digital communications [18,19].

Notwithstanding the elaborate, but empirical studies, optimal selection of a filtering strategy for state estimation in a non-linear system is still an open problem. The problems associated with non-linear filter selection are highlighted in several recent studies. For instance, [14] reported that EKF and UKF performed better than EnKF in a batch crystallizer and [20] reported better EnKF performance compared to EKF and UKF on a packed bed reactor. Similarly, [21] reported that SIR outperformed UKF in estimating the states of a gas-phase reaction in a continuous stirred-tank reactor, but [15] showed that UKF outperformed SIR on a polymer reactor. Similar inconclusive results have also been observed with other pairs of non-linear filters, such as EnKF and SIR (see [12,22]), and SIR and ASIR (see [23,24]). Apart from these results, a recent study done on a stochastic Lorenz system suggests that it is possible for two filters to outperform each other in different operating regions. For example, the authors in [25] showed that the performance of EnKF is better than that of SIR, and vice versa depending on the operation region. This highlights the sensitivity of the performance of a non-linear filter to the process dynamics, the filter approximation and the process operating conditions.

The elaborate empirical studies done on non-linear filters, suggest that, there is no single non-linear filter that provides the best state estimates for any given process. Moreover, it is not even possible to choose a non-linear filter that retains high filtering performance on a given process under all operating conditions. A practitioner is thus left with no clear substitute for the optimal non-linear filter. An approach to resolve this dilemma is to start with a family of non-linear filters and switch between them, as and when required, so as to maintain high filtering performance. Naturally, this approach has to depend on the performance of non-linear filters. The filtering performance of a non-linear filter is a function of the process dynamics and operating conditions – two properties over which a user has limited leverage. Accurate characterization and quantification of filtering performance of a filter is thus crucial, as it helps in:

- *Performance assessment*
-Comparison of the filtering performance of a non-linear filter against that of an optimal non-linear filter.
- *Performance diagnosis*
-Study of the effects of the underlying numerical and statistical approximations in a non-linear filter on the quality of the state estimates obtained therefrom.
- *Non-linear filter selection*
-Construction of an effective filtering strategy for state estimation in a given non-linear process.

Despite the strong practical interest in evaluating the performance of non-linear filters, it remains one of the most complex problems in non-linear state estimation [26]. Several authors – [16,27,28] – proposed using the normalized estimation error squared (NEES) as a non-linear filter performance metric. When the state estimation error for a non-linear filter is approximately Gaussian, it can be shown that the NEES follows a Chi-square distribution with certain degrees of freedom [27]. The performance of a filter is then judged based on the hypothesis test-based confidence levels constructed using the NEES values. Despite its simplicity, the NEES computations require a priori knowledge of the true states. The true states are generally not available, except in computer simulations or in carefully conducted experiments [29]. [30] proposed replacing the NEES with normalized innovation square (NIS); however, the metric is not particularly meaningful for multimodal measurement models. Moreover, since the NIS is based on the measurements and not the states, inference based on NIS alone is not reliable [31]. The authors in [32] proposed a metric based on the degree of process non-linearity – measured using the magnitudes of the first and second-order terms in a Taylor series approximation of a non-linear process – which was exploited by [33,34] in assessing the performance of various non-linear filters.

The NEES, NIS and other non-linearity-based metrics provide a convenient and quick way to qualitatively assess the performance of a non-linear filter in real-time; however, there are limitations of these metrics as summarized in [27,35]: (i) fail to provide any quantitative measure of filter performance; (ii) not useful in comparing multiple filters; (iii) computation requires a priori access to the true states; (iv) can be constructed and computed only for certain classes of non-linear SSMs; (v) depend on the input-output data; (vi) require the state estimation error and innovation sequence to have a Gaussian distribution; and (vii) provide limited insights on performance diagnosis.

A performance metric that improves on the weaknesses of NEES, NIS and other non-linearity-based metrics is therefore crucial for assessment, diagnosis and selection of non-linear filters. The conventional Cramér–Rao lower bound (CRLB) inequality provides a theoretical lower bound on the mean square error (MSE) of any maximum-likelihood (ML) based unbiased state or parameter estimator. An analogous extension of the CRLB inequality to the Bayesian estimators was derived by [36] and is called the PCRLB inequality. The PCRLB inequality is general, and provides a lower bound on the MSE of a non-linear filter [36,37]. Furthermore, the lower bound only depends on the fundamental properties of the process – the process dynamics, choice of prior density for the states, and noise characteristics [38]. The lower bound or the PCRLB is in fact independent of the choice of a non-linear filter or any particular realization of the input-output data. Practical applications of the PCRLB inequality, include: comparison of several non-linear filters for ballistic target tracking [16], terrain navigation [39], and design of systems with pre-specified performance bounds [40]. Unlike other metrics, the PCRLB inequality can only be computed off-line. Nevertheless, for many real-time applications of state estimation (e.g., control and process monitoring) – the design, performance evaluation, diagnosis and selection of non-linear filters are mostly done a priori or off-line. The PCRLB inequality, thus provides a reliable alternative to other previously reported metrics. Despite the possibility, use of a PCRLB inequality-based metric in performance assessment, diagnosis and selection of non-linear filters remains largely unexplored, and is the focus of this paper. The contributions of this paper are discussed next.

2. Contributions

The following are the main contributions:

- A PCRLB inequality-based metric is proposed for off-line performance assessment of multiple non-linear filters.