ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Fully discrete stabilized multiphysics finite element method for the polymer gel model☆

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ARTICLE INFO

Article history:
Received 20 October 2017
Received in revised form 16 April 2018
Accepted 20 April 2018
Available online xxxx

Keywords:
Polymer gel model
Multiphysics finite element method
Stabilized method

ABSTRACT

In this paper, we propose a fully discrete stabilized multiphysics finite element method by the pressure projection method to solve the polymer gel model. To reveal the multiphysical process of the polymer gel model, we reformulate the polymer gel model by introducing a new variable to get a new multi-field problem, then we propose the fully discrete stabilized multiphysics finite element method using the simplest equal order element of $P_1-P_1-P_1$ without the discrete inf–sup condition to solve the new reformulated system, and we prove that the new method not only has no "locking" phenomenon, but also preserves some energy conservation laws. Also, we prove that the stabilized multiphysics finite element method has optimal convergence order. Finally, we give the numerical example to verify the theoretical results.

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1. Introduction

A gel is a soft poroelastic material which consists of a solid network and a colloidal solvent. In general, gels are apparently solid, jelly-like materials, and play an important role in many areas such as biomedical and chemical systems, for more details one can see [1–5] and the references therein. Since the polymer gel model is very complicated, so there are few results both in theory and computation. As for the numerical results, a mixed finite element method by introducing an elastic pressure and decoupling the polymer gel model into two subproblems for the polymer gel model is proposed by [6] using $P_2-P_1-P_1$ element pair. We note that the mixed finite element method for $P_2-P_1-P_1$ element pair not only costs huge computation, but also it is not easy to be implemented on computer. To reduce the computation, reveal the multiphysics process of the gel model and overcome the "locking" phenomenon (as for the detail of "locking" phenomenon, one can see [7]), we use the lowest equal order element pairs of $P_1-P_1-P_1$ to solve the polymer gel model. However, it is well-known that the element pairs of $P_1-P_1-P_1$ do not satisfy the discrete inf–sup condition. Due to the facts of the simple formula and easily practical importance in scientific computation, the element pair of $P_1-P_1-P_1$ is very attractive if the numerical method is handled with some stabilized technique. In the paper, we introduce the pressure projection method following the idea of [8] to get a stabilized multiphysics finite element method by the pressure projection method. About the similar stabilized methods for other problems such as Stokes equations and Navier–Stokes equations, one can refer to [9–13] and so on. As for the other stabilized methods without the residuals, one can see [14–17] and the references therein. To our knowledge, it is the first time to propose the stabilized multiphysics finite element method for the polymer gel model.

☆ The work is supported by the Natural Science Foundation of Henan Province (No.: 162300410031), and the Excellent Youth Program of Henan University (No.: yqpy20140039).

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https://doi.org/10.1016/j.camwa.2018.04.025
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Please cite this article in press as: Z. Ge, et al., Fully discrete stabilized multiphysics finite element method for the polymer gel model, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.04.025.

The remainder of this paper is organized as follows. In Section 2, we introduce the notations and some basic results. In Section 3, we propose the fully discrete stabilized multiphysics finite element methods for $P_1-P_1-P_1$ element pairs without the discrete inf–sup condition. And we prove that the fully discrete multiphysics finite element method is not only stable, but also satisfies some discrete energy conservation laws. In Section 4, we show that the convergent order of the error is optimal. Finally, we give the numerical example to show that our method is effective, and the numerical method has great advantages than one of [6] in the cases of the cost of computation and the stability of the numerical solution. Moreover, we also demonstrate that our method can avoid the "locking" phenomenon.

2. Polymer gel model and some basic results

In this paper, we study the following displacement–pressure model for the polymer gel, which is given by [1,3]:

$$(\alpha+\frac{\beta}{2})\nabla \mathrm{div}(\mathbf{u})+\frac{\beta}{2}\Delta\mathbf{u}=\nabla p \ \text{ in } \ \Omega_T, \tag{2.1}$$

$$\mathrm{div}\mathbf{u}_t=\kappa\Delta p, \ \ \kappa:=\frac{(1-\phi)^2}{\xi} \ \text{ in } \ \Omega_T, \tag{2.2}$$

$$\widehat{\sigma}(\mathbf{u},p)\nu:=(\sigma(\mathbf{u})-pI)\nu=f, \ \ \frac{\partial p}{\partial \nu}=0 \ \text{ on } \ \partial\Omega_T, \tag{2.3}$$

$$\mathbf{u}(x,0)=\mathbf{u}_0(x) \ \text{ on } \ \Omega, \tag{2.4}$$

where the space–time domain $\Omega_T=\Omega(\subset\mathbb{R}^d, d=2,3)\times(0,T)$ and $\partial\Omega_T=\partial\Omega\times(0,T)$ for some given $T>0$, and $\sigma(\mathbf{u})$ stands for the stress tensor of the gel network, given by $\sigma(\mathbf{u})=\alpha\mathrm{div}\mathbf{u}I+\beta\varepsilon(\mathbf{u})$, where $\varepsilon(\mathbf{u})=\frac{1}{2}(\nabla\mathbf{u}+\nabla\mathbf{u}^T)$, α and β are Láme constants, ϕ is the volume fraction of the polymer, ξ is the friction constant associated with the motion of the polymer relative to the solvent, ν denotes the outward normal to $\partial\Omega$, and I denotes the identity matrix, $\mathbf{f}$ is a given vector function.

To reformulate the problem (2.1)–(2.4), we introduce $q=\mathrm{div}\mathbf{u}$, which is called elastic pressure, to get the following new problem:

$$\beta\mathrm{div}(\varepsilon(\mathbf{u}))=\nabla\widetilde{p}, \ \ \widetilde{p}:=p-\alpha q, \tag{2.5}$$

$$\mathrm{div}\mathbf{u}=q, \tag{2.6}$$

$$q_t=\kappa\Delta p, \ \ p=\widetilde{p}+\alpha q, \tag{2.7}$$

$$\widehat{\sigma}(\mathbf{u},p)\nu=(\sigma(\mathbf{u})-pI)\nu=\mathbf{f}, \ \ \frac{\partial p}{\partial\nu}=0 \ \text{ on } \ \partial\Omega_T, \tag{2.8}$$

$$\mathbf{u}(\cdot,0)=\mathbf{u}_0(\cdot) \ \text{ in } \ \Omega. \tag{2.9}$$

The problem (2.5)–(2.9) reveals the underlying deformation and diffusion multi-physical process which occur in swelling dynamics of the gel, that is, we can decouple the problem (2.5)–(2.9) by a splitting time-stepping algorithm, i.e., $(\mathbf{u},\widetilde{p})$ satisfies a generalized Stokes problem and q satisfies a diffusion problem.

To define the weak solution of the problem (2.5)–(2.9), we introduce the following function spaces:

$$L_0^2(\Omega):=\{q\in L^2(\Omega):\int_\Omega q dx=0\}, \quad \mathbf{X}:=\mathbf{H}^1(\Omega)=[H^1(\Omega)]^d.$$

Furthermore, the space $L^2(\Omega)$ is endowed with the L^2-scalar product and L^2-norm denoted by $(\cdot,\cdot)$ and $\|\cdot\|_0$. The space $\mathbf{X}$ is equipped with its usual scalar product and norm $((u,v))=(\nabla u,\nabla v), \quad \|u\|_1=((u,u))^{1/2}$.

Define

$$\mathbf{RM}=\{\mathbf{r}\in\mathbf{H}^1(\Omega):\varepsilon(\mathbf{r})=0\}=\{\mathbf{r}:=\mathbf{a}+\mathbf{b}\times x:\mathbf{a},\mathbf{b},x\in\mathbb{R}^d\},$$

where ε is the strain operator, and $\mathbf{RM}$ is the space of infinitesimal rigid motion.

Let $\mathbf{H}_\perp^1(\Omega)$ denote the subspace of $\mathbf{H}^1(\Omega)$, which is orthogonal to $\mathbf{RM}$, i.e.

$$\mathbf{H}_\perp^1(\Omega)=\{\mathbf{v}\in\mathbf{H}^1(\Omega):(\mathbf{v},\mathbf{r})=0 \ \ \forall\mathbf{r}\in\mathbf{RM}\}.$$

From [6,18], we know that there exists a positive constant c such that

$$\|\mathbf{v}\|_1\le c\|\varepsilon(\mathbf{v})\|_0 \ \ \forall\mathbf{v}\in\mathbf{H}_\perp^1(\Omega).$$

Also, we introduce the following space:

$$\mathbf{L}_\perp^2(\partial\Omega):=\{\mathbf{g}\in L^2(\partial\Omega):\langle\mathbf{g},\mathbf{v}\rangle=0 \ \ \forall\mathbf{v}\in\mathbf{RM}\}.$$

Throughout the paper, we assume that Ω is a bounded polygonal domain such that $\Delta:H_0^1(\Omega)\cap H^2(\Omega)\to L^2(\Omega)$ is an isomorphism (see [19]). In addition, we use C to denote a generic positive constant whose value may change from place to place but that remains independent of the mesh parameter h and Δt and the solution of the problem.

Please cite this article in press as: Z. Ge, et al., Fully discrete stabilized multiphysics finite element method for the polymer gel model, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.04.025.

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