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Set-based adaptive estimation for a class of nonlinear systems with time-varying parameters

Samandeep Dahliwal [a], Martin Guay [b,*]

[a] Hatch and Associates, Sudbury, ON, Canada
[b] Department of Chemical Engineering, Queen's University, Kingston, ON, Canada

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ABSTRACT

In this paper, an adaptive estimation technique is proposed for the estimation of time-varying parameters for a class of continuous-time nonlinear system. A set-based adaptive estimation is used to estimate the time-varying parameters along with an uncertainty set. The proposed method is such that the uncertainty set update is guaranteed to contain the true value of the parameters. Unlike existing techniques that rely on the use of polynomial approximations of the time-varying behaviour of the parameters, the proposed technique does require a functional representation of the time-varying behaviour of the parameter estimates. A simulation example and a building systems estimation example are considered to illustrate the developed procedure and ascertain the theoretical results.

1. Introduction

The problem of parameter estimation has been of considerable interest during the last two decades. The vast majority of the literature on adaptive estimation and adaptive control relies on the assumption that the parameters of the system are slowly time-varying or constant. In practice however, the time-varying behaviour of the process parameters may be of significant importance. Time-varying parameters can arise from uncertain complex mechanisms not captured by the process model. They can also arise from model-plant mismatch or unmeasured inputs that affect the process dynamics.

Evidently, the ability to estimate such time-varying behaviour can have a significant impact on control system performance. Several researchers have considered the problem of estimation of time-varying parameters. In [18], an estimation routine is proposed for a class of nonlinear systems with time-varying parameters. A Taylor series expansion of the time-varying parameters is considered. The time varying behaviour is captured by applying a local polynomial expansion and estimating the (locally) constant Taylor series coefficients using a standard least-squares approach. The authors demonstrate the convergence of the parameter estimates to a neighbourhood of the true parameters within a given resetting period. An related approach is proposed in [5] where a direct least-squares based method is used to estimate the coefficients of a local polynomial expansion of the time-varying parameters. In [7], a sliding mode approach is proposed where convergence of the parameter estimates to the true estimates is guaranteed. The technique relies on an estimate of the sign of the parameter estimation error which may be very difficult to obtain in practice. An alternative approach was proposed in [12] for the estimation of time-varying parameters using a dynamic inversion approach. In this approach, it is shown that inversion technique can be useful in the design of reliable parameter estimation techniques subject to the assumption that the regressor vector is uniformly full rank.

Many researchers have considered the problem of unknown input estimation which is very closely related to problem of estimation of time-varying parameters. Combined state and unknown input estimation for a class of nonlinear systems is proposed in [6]. High-gain observer methods are proposed in [13]. A sliding-mode observer design approach is proposed in [14] for the estimation of unknown inputs. In contrast to the estimation of time-varying parameters, the estimation of unknown inputs rely on often restrictive structural assumptions which are related to the observability of the inputs from the available measurement. In parameter estimation, this structural information would in general be limited to an identifiability requirement or a persistency of excitation condition.

The analysis of nonlinear systems with time-varying parameters has been considered in [3]. In this paper, the authors propose an adaptive observer canonical form and propose an adaptive observer or this class of systems. It is claimed that the estimation of time-varying parameters can be performed subject to a bound on the rate of change of the parameters. A related result is presented in [10] where an adaptive observer is proposed for a class of nonlinear

* Corresponding author. Tel.: +1 613 533 2788; fax: +1 613 533 6637.
E-mail address: martin.guay@chee.queensu.ca (M. Guay).

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systems with time-varying parameters and bounded disturbances. As in [3], estimation of the time-varying parameters is possible subject to a bound on the derivative of the parameters.

The present paper is a generalization of the identification scheme presented in [2] to the problem of time-varying parameter estimation. We propose a set-based parameter estimation technique that provides simultaneously estimates of the parameters along with an estimate of the uncertainty of the parameters. The approach provides conditions that ensure the convergence of the parameter to a neighbourhood of the true value. In addition, we develop a set-update algorithm that updates the uncertainty set to guarantee non-exclusion of the true parameter estimate.

This paper is organized as follows. The problem description is given in Section 2. The parameter estimation routine and uncertainty set adaptation are presented in Section 3. Two simulation examples are presented in Section 4 followed by brief conclusions in Section 5.

2. Problem description

Consider a nonlinear system

$$\dot{x} = f(x) + \sum_{i=1}^{p} g_i(x)\theta_p(t) = f(x) + g(x)\theta(t) \tag{1}$$

where $f(x)$ and $g_i(x)$ ($i = 1, \ldots, n$) are sufficiently smooth vector valued functions, $g(x) = [g_1(x), \ldots, g_p(x)] \in \mathbb{R}^{n\times p}$, $x \in \mathbb{R}^n$ is the state, $\theta(t) \in \mathbb{R}^p$ is an unknown time varying bounded parameter vector. The following assumptions are made about (1).

Assumption 1. The state trajectories $x(t)$ evolve on a compact subset $\mathbb{X}$ of $\mathbb{R}^n$.

Assumption 2. The unknown parameter vector $\theta(t)$ is such that $\|\dot{\theta}(t)\| < c_1$, where c_1 is a positive constant.

Assumption 3. The parameter vector $\theta(t)$ is assumed to be uniquely identifiable and lies in a known compact set $\Theta^0 = B(\theta_0, z_\theta)$, where $\theta_0 = \theta(0)$ is a nominal parameter value and $z_\theta > 0$ is the radius of the parameter uncertainty set.

Remark 4. Identifiability is a standard requirement in adaptive estimation. It states that for any two output trajectories $y(t, \theta_1)$ and $y(t, \theta_2)$ corresponding to two parameter values θ_1 and θ_2, respectively, the following property can be guaranteed:

$$y(t, \theta_1) = y(t, \theta_2), \quad \forall t \ge t_0 \Rightarrow \theta_1 = \theta_2.$$

Assumption 5. The state variables are available for measurement in continuous time.

The objective of this paper is provide an estimation of the unknown time-varying parameters along with an estimate of their uncertainty.

Remark 6. The class of nonlinear system considered does not explicitly take into account the effect of measurement noise and disturbances. However, the time-varying parameters can be interpreted as process disturbances that are subject to a bound on their time derivative. If the disturbances are such that they do not meet Assumption 3 then one must assume that the disturbances have a known bound as in [2]. The implication is that the disturbances will increase the size of the parameter uncertainty region.

In situations where the system is subject to measurement noise, one must write the measured state variables as $x = z + \omega(t)$ where $\omega(t)$ is a bounded signal with zero mean. The dynamics (1) must be interpreted in terms of the noisy state measurement x. That is, it is assumed that one can write:

$$\dot{x} = \dot{z} + \dot{\omega} = f(x) + g(x)\theta(t) + \nu(t)$$

where $\nu(t)$ is a disturbance that depends on the dynamics of the measurement noise $\omega(t)$ and the difference between the internal state z and its measurement x. This term can be added as a parameter to be estimated if it meets the Assumptions stated above. In most situations, one can assume that the measurement noise ω is small relative to the magnitude of the state measurement, i.e., $z(t) \approx x(t)$.

If, on the other hand, the system is stated in terms of the internal state variables:

$$\dot{z} = f(z) + g(z)\theta(t)$$
$$x = z + \omega$$

then an adaptive filter for the estimation of z and θ is required. The design of such nonlinear filters is beyond the scope of the proposed approach.

3. Parameter and uncertainty set estimation

3.1. Parameter estimation

Let K be an n by n matrix chosen such that $K + K^T > I$. Let the state predictor model for (1) be chosen as

$$\dot{\hat{x}} = f(x) + g(x)\hat{\theta}(t) + Ke + w^T\dot{\hat{\theta}}(t), \tag{2}$$

$$\dot{w}^T = -Kw^T + g(x), \quad w(t_0) = 0. \tag{3}$$

where $\hat{x} \in \mathbb{R}^n$ is the vector of predicted state variables, $w \in \mathbb{R}^{n\times p}$ is the regression matrix, $e = x - \hat{x}$ is the state estimation error, $\hat{\theta} \in \mathbb{R}^p$ are the parameter estimates and $\dot{\hat{\theta}}$ is the parameter estimation update law to be defined below. By (2) and (3), the error dynamics are given by:

$$\dot{e} = g(x)\tilde{\theta}(t) - Ke - w^T\dot{\hat{\theta}}(t) \tag{4}$$

Next, we define an auxiliary variable $\eta = e - w^T\tilde{\theta}(t)$. The η dynamics are given by:

$$\dot{\eta} = -K\eta - w^T\dot{\theta}(t), \quad \eta(t_0) = e(t_0) \tag{5}$$

Since the rate of change of the unknown parameter $\dot{\theta}$, we define an estimate of η with dynamics given by:

$$\dot{\hat{\eta}} = -K\hat{\eta} \tag{6}$$

We define the η estimation error $\tilde{\eta} = \eta - \hat{\eta}$ with dynamics

$$\dot{\tilde{\eta}} = -K\tilde{\eta} - w^T\dot{\theta}(t), \quad \tilde{\eta}(t_0) = 0. \tag{7}$$

In the following, a bound on $\|\tilde{\eta}\|$ is required. Such a bound can be generated as follows.

Let us first note that the initial estimate $\tilde{\eta}(0) = \eta(0) - \hat{\eta}(0) = 0$. Next, consider the Lyapunov function $V_\eta = (1/2)\tilde{\eta}^T\tilde{\eta}$. The rate of change of V_η is given by:

$$\begin{aligned}\dot{V}_\eta &= \tilde{\eta}^T\left[K\tilde{\eta} - w^T\dot{\theta}(t)\right] = -\tilde{\eta}^TK\tilde{\eta} + \tilde{\eta}^Tw^T\dot{\theta} \\ &\le -\tilde{\eta}^TK\tilde{\eta} + \frac{k_1}{2}\tilde{\eta}^Tww^T\tilde{\eta} + \frac{1}{2k_1}\dot{\theta}^T\dot{\theta}\end{aligned} \tag{8}$$

where k_1 is some positive constant.

Thus if one assigns K as

$$K = \frac{k_2}{2}I + \frac{k_1}{2}ww^T$$

for some positive constant k_2 with I, the identity matrix, the inequality above becomes:

$$\dot{V}_\eta \le -k_2V_\eta + \frac{1}{2k_1}\|\dot{\theta}\|^2 \le -k_2V_\eta + \frac{c_1^2}{2k_1} \tag{9}$$