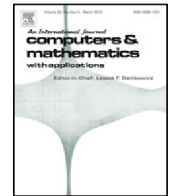




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Existence results and iterative method for solving a nonlinear biharmonic equation of Kirchhoff type

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ABSTRACT

In this paper, for investigating a boundary value problem for a nonlinear biharmonic equation of Kirchhoff type, we propose an efficient approach by the reduction of the problem to an operator equation for the nonlinear part of the equation. The result is that we have established the existence and uniqueness of a solution under some easily verified conditions. Also, we propose an iterative method for finding the solution. Some examples demonstrate the applicability of the theoretical results and the efficiency of the iterative method.

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1. Introduction

In the paper, we consider the following nonlinear biharmonic boundary value problem (BVP):

$$\begin{aligned} \Delta^2 u &= M\left(\int_{\Omega} |\nabla u|^2 dx\right) \Delta u + f(x, u), \quad x \in \Omega, \\ u &= 0, \quad \Delta u = 0, \quad x \in \Gamma, \end{aligned} \quad (1.1)$$

where Ω is a connected bounded domain in \mathbb{R}^K ($K \geq 2$) with a smooth boundary Γ , Δ is the Laplace operator, Δ^2 is the biharmonic operator, ∇u is the gradient of u , $f: \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ and $M: \mathbb{R}^+ \rightarrow \mathbb{R}$ are continuous functions. This problem describes the nonlinear static deflection of an elastic plate.

In one-dimensional case ($K = 1$), by using variational methods and some fixed point theorems in cones, the existence and multiplicity results for the problem for Eq. (1.1) with other boundary conditions are considered (see e.g. [1–4]). For a particular case, when $M(t) = t$ and $f = f(x)$ in [5] the authors studied the existence and uniqueness of a solution and proposed an iterative method for finding the solution.

In the multi-dimensional case $K > 1$, also by using the variational methods, many authors studied the existence and multiplicity of nontrivial solutions of the problem (1.1) (in the both cases when M is a constant or not) (see e.g. [6–10]). It should be noticed that in the listed works, the function $f(x, u)$ is assumed with very strong assumptions, and there are no examples of solutions to verify theoretical results of their existence. Below we mention two typical works of them.

The first work is of Hu and Wang [7] concerned with the case when M is a constant function, namely, the problem

$$\begin{aligned} \Delta^2 u + c \Delta u &= f(x, u), \quad x \in \Omega, \\ u &= 0, \quad \Delta u = 0, \quad x \in \Gamma, \end{aligned}$$

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