Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Higher-order regularization based image restoration with automatic regularization parameter selection

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ARTICLE INFO

Article history:
Received 29 October 2015
Received in revised form 31 December 2017
Accepted 8 April 2018
Available online xxxx

Keywords:
Poisson denoising
Multiplicative noise removal
Rayleigh noise
Higher-order regularization
Spatially adaptive regularization parameter

ABSTRACT

Poisson and multiplicative Rayleigh noises often appear in medical imaging such as X-ray images, positron emission tomography, and ultrasound images. In this study, we propose novel variational models for removing Poisson/multiplicative Rayleigh noise. We make use of hybrid higher-order total variation as the regularization terms of our proposed models to eliminate staircasing artifacts. We also adopt the spatially adaptive parameter technique to adequately smooth homogenous regions while preserving the edges. The spatially adaptive parameter selection is closely related to local constraints through a local expected value estimator. We provide a convergence analysis, including the existence and uniqueness of solution, and the first order optimality conditions. We apply the alternating direction method of multipliers for solving the proposed models. Numerical experiments demonstrate that our models exhibit a better performance than that of state-of-the-art models in terms of edge preservation, smoothness of the homogenous regions, and various quality measures.

1. Introduction

In several real-world applications, images are frequently contaminated by noise owing to the image acquisition process or naturally occurring phenomena. Therefore, it is important to realize image denoising in image processing. The goal of image denoising is to remove noises from a given corrupted digital image while maintaining essential features such as edges and textures. In this article, we focus on the removal of two types of non-Gaussian and non-additive noise, i.e., Poisson noise and multiplicative Rayleigh noise.

The Poisson noise appears in several imaging systems, such as positron emission tomography [1], radiography [2], fluorescence microscopy [3], optical nanoscopy [4] and charge-coupled-device (CCD) cameras [5]. Generally, the degradation model [6] for the observed data f degraded by Poisson noise is given by

$$f = \frac{1}{\rho} Poisson(\rho \cdot u),$$

where $u : \Omega \to \mathbb{R}$ is a clean image in a bounded, connected, and open domain $\Omega \subset \mathbb{R}^2$ with a Lipschitz boundary, $\rho > 0$ is a constant that expresses the degree of image degradation, and $Poisson(k)$ is a function that returns Poisson random numbers for the mean k of the Poisson distribution. The Poisson distribution [7] in terms of the mean parameter k can be defined as

$$P(x\,;\,k) = \frac{e^{-k}k^x}{x!}, \quad x \geq 0.$$

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https://doi.org/10.1016/j.camwa.2018.04.004

The Poisson noise is signal-dependent, and thus, the removal of Poisson noise is more difficult than that of Gaussian noise, impulse noise, and other types of signal-independent noise. As in the early work of Richardson and Lucy [8,9], various methods have been proposed for removing Poisson noise [10–12]. In a variational framework, Le et al. [13] proposed a minimization problem based on the total variation (TV) regularizer [14]. Despite the edge-preserving property of TV, several variational models with different regularization terms have been proposed, such as Schatten-norm [15], tight frame [16], total generalized variation [17], and higher order TV [18] regularization, in order to overcome the staircasing artifacts of TV. Moreover, to improve the restoration quality, the spatially adapted regularization parameter (SARP) selection [19] was utilized for the TV-based Poisson image restoration model. In this work, we propose a convex variational model that incorporates the higher-order TV regularizer with the SARP approach for the removal of Poisson noise.

The multiplicative noise occurs often in a coherent imaging system, such as synthetic aperture radar (SAR), ultrasound (US) imaging, sonar and laser imaging. The degradation model of the observed data f that is corrupted by the multiplicative noise is given by

$$f = u \cdot n, \tag{1}$$

where u is a clean image, and n is a noise that follows a distribution such as a Gamma distribution or Rayleigh distribution. In this work, we consider the multiplicative Rayleigh noise present in US images in particular. The probability density function of the Rayleigh distribution [20] is given by

$$P(x;\sigma) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right), \quad x \geq 0,$$

where $\sigma > 0$ is a scale parameter of the Rayleigh distribution.

The multiplicative noise is also a signal-dependent noise, and thus, it is challenging to remove the multiplicative noise from the observed data in contrast to the case of additive noise. In order to remove the multiplicative noise in US images, a variety of filtering-based methods have been proposed [21–26]. However, there are few variational models for multiplicative Rayleigh noise removal. The first variational model is a TV-based minimization model [27] with applications to SAR images. Recently, a new variational model [28] based on a nonconvex TV regularizer and tight framelet was proposed. Moreover, a convex TV-based variational model was proposed in [29], which could handle the general multiplicative noise. In contrast to the aforementioned TV-based models, we use a higher-order TV regularization for the multiplicative Rayleigh noise removal and make use of the SARP approach to further enhance the restoration quality in our model.

The general variational model comprises an objective function consisting of a data fidelity term and a regularization term. The data fidelity term measures the distance between a clean image u and degraded image f, i.e., this term has a minimum value at $u = f$ if it is convex and smooth. The regularization term causes the variational model to be well-posed and enables the removal of noise in the degraded image. Various regularizers have been proposed in the extant studies. The TV regularizer [14] is popular and can preserve edges well in the output image. However, the smooth regions in restored images often look like stair and these artifacts are often called “staircasing artifacts”. In order to cope with the staircasing artifacts, a higher-order TV was proposed as a regularization term in [30]. However, the use of the second-order TV results in less preserved edges than those obtained with TV. Some prior works have been conducted to integrate TV with higher-order regularization in order to reduce the staircasing artifacts and preserve edges. One such work involves the convex combination of TV and second-order TV as regularization terms [31,32]. Another such approach makes use of the inf-convolution [33] of TV and higher-order TV. As a generalization of the inf-convolution, the total generalized variation (TGV) was proposed by Bredies, Kunisch and Pock [34].

In a variational framework, we require a regularization parameter that balances the data fidelity term and smoothing term, which controls the smoothness of a restored image. If we include the regularization parameter in the data fidelity term, a large value of the regularization parameter results in leftover noise in the homogenous regions, while a small value results in an oversmoothed restored image. Therefore, we expect that a small value of the regularization parameter in homogenous parts and a large regularization parameter in the textural parts would yield a better restored image than that obtained when using a constant regularization parameter. Therefore, the SARP approaches have been proposed in several prior works [35–41] for various image processing problems. A SARP approach for Poisson noise removal was proposed in [19] based on the approaches in [37] using theoretical analysis. Moreover, the SARP methods for multiplicative Gamma noise removal were employed in several works [38,39]. However, no study has provided a SARP strategy for multiplicative Rayleigh noise removal. In this work, we propose new SARP approaches for the removal of Poisson or multiplicative Rayleigh noise.

In this paper, we propose new variational models for the restoration of images that are corrupted by Poisson or multiplicative Rayleigh noise. In order to reduce the noise efficiently and retain the image details, we develop a novel strategy for the automated selection of the SARP. We also make use of a combination of TV and higher order TV as regularization terms. This facilitates the effective alleviation of staircasing artifacts in the restored images. We present a convergence analysis involving the existence and uniqueness of solutions and the first-order optimality characterization. We also provide efficient numerical algorithms for our proposed models.

This paper is organized as follows. In Section 2, we review the bounded variation (BV), bounded Hessian (BH) spaces and their properties. We also present several variational models for the Poisson or multiplicative Rayleigh noise removal. In Section 3, we propose variational models that involve a hybrid higher-order TV regularization and a SARP technique.

Please cite this article in press as: M. Kang, et al., Higher-order regularization based image restoration with automatic regularization parameter selection, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.04.004.