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LU-based Jacobi-like algorithms for non-orthogonal joint diagonalization

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ABSTRACT

In this paper, based on the LU decomposition, we propose three non-orthogonal Jacobi-like alternating iterative algorithms with two strategies for solving the joint diagonalization problem of a set of Hermitian matrices. In this kind of algorithm, each transformation includes one upper triangular iterative step and one lower triangular iterative step, and each step involves one parameter. The optimal parameter of each step is derived analytically. The convergence of our proposed algorithms is proven. According to this convergence analysis, the existing GNJD algorithm is revisited. Finally, numerical simulations are presented to illustrate the effectiveness of the proposed algorithms in comparison with existing ones.

1. Introduction

In recent years, the joint diagonalization (JD) problem of a set of matrices has attracted more and more attention because it is instrumental in a number of signal processing problems as in independent component analysis (ICA) [1], array signal processing [2] and more significantly, blind source separation (BSS) [3–11].

A number of algorithms have been proposed to solve the JD problem. For example, the ARD algorithm was established by Wang, Liu and Zhang in [12]. This algorithm alternates the update of each row of the de-mixing matrix, and the update of each row needs to solve the eigenvector corresponding to the minimal eigenvalue of an Hermitian positive (semi-)definite matrix. The FAJD algorithm was proposed by Li and Zhang [13]. It added a penalty function to the quadratic criterion to avoid degenerate solution or trivial solution generated by the ARD algorithm. The FFDIAG algorithm was developed by Ziehe and Muller [14] and the de-mixing matrix is decomposed as the product of a series of strictly diagonal dominant matrices.

In the spirit of FFDIAG, but based on an LU matrix decomposition, useful algorithms have been proposed. In [15], Afsari decomposed the diagonalizing matrix into its LU form in the real case, and proposed the LUJ1D algorithm. In [16], Wang, Gong and Lin extended the LUJ1D algorithm to the complex case and established the LUCJD algorithm. In [17], Maurandi and Moreau proposed the ALUJA algorithm based on both a special parameterization of the diagonalizing matrix and an adapted local criterion. In [18], Gong, Wang and Lin extended the LUCJD algorithm to solve the problem of the multiple dataset, and proposed the GNJD algorithm. Both the GNJD algorithm and the LUJ1D algorithm compute the optimal entries first in the upper triangular part and then successively in the lower triangular part of the updating matrix at each iteration.

The main contributions of this paper are as follows. In contrary to the above algorithms, a first main point of our proposal is to alternatively handle the optimal entries both at coupled symmetric positions concerning the updating matrix at each iteration. Moreover, by taking advantage of the LU decomposition within the general framework of Jacobi-like algorithms

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that are originally intended for solving a two dimensional linear system, we propose three algorithms. Finally, a convergence analysis is derived that allows to reconsider the GNJD algorithm.

The remainder of this paper is organized as follows. In Section 2, we formulate the JD problem and present the principles of our algorithms. In Section 3, the optimal updating matrix is derived. In Section 4, the convergence analysis is presented. In Section 5, numerical simulations are given to illustrate the performance of the proposed algorithms. Finally, a conclusion is drawn in Section 6.

2. Problem formulation and algorithm principle

We consider the joint matrix diagonalization problem. Suppose that we have a set of Hermitian matrices $\{\mathbf{C}_k \in \mathbb{C}^{N\times N} : k = 1, 2, \ldots, K\}$ (associated with some observed signals) that all share the following common decomposition:

$$\mathbf{C}_k = \mathbf{A}\mathbf{D}_k\mathbf{A}^H + \mathbf{N}_k, \tag{1}$$

where the superscript $(.)^H$ denotes the conjugate transpose, $\mathbf{A} \in \mathbb{C}^{N\times N}$ denotes a non-singular mixing matrix and $\mathbf{D}_k, \mathbf{N}_k \in \mathbb{C}^{N\times N}$ are diagonal matrices and additive noise matrices for all $k = 1, \ldots, K$, respectively. The aim of the JD problem is to estimate a non-singular "diagonalizing" matrix $\mathbf{B} \in \mathbb{C}^{N\times N}$ such that all $\mathbf{B}\mathbf{C}_1\mathbf{B}^H, \mathbf{B}\mathbf{C}_2\mathbf{B}^H, \ldots, \mathbf{B}\mathbf{C}_K\mathbf{B}^H$ are as diagonal as possible. The matrix $\mathbf{B}$ is often referred to as "de-mixing" matrix in the context of BSS.

In order to measure the diagonality of the transformed matrices, we use the following cost function:

$$\mathcal{J}(\mathbf{B}) = \sum_{k=1}^{K} \mathrm{off}(\mathbf{B}\mathbf{C}_k\mathbf{B}^H), \tag{2}$$

where $\mathrm{off}(\mathbf{X})$ is the sum of all off-diagonal entries squared of the matrix $\mathbf{X}$.

In the same spirit as [5,6] and [18], the first goal of this paper is to establish three algorithms based on two different strategies. For that, we propose to carry out the estimation of $\mathbf{B}$ by using two successive multiplicative updates as

$$\begin{cases} \mathbf{B}^{(t+\frac{1}{2})} = (\mathbf{I} + \mathbf{U}^{(t)})\mathbf{B}^{(t)}, \\ \mathbf{B}^{(t+1)} = (\mathbf{I} + \mathbf{L}^{(t)})\mathbf{B}^{(t+\frac{1}{2})}, \end{cases}$$

where $\mathbf{L}^{(0)}$, $\mathbf{U}^{(0)}$ and $\mathbf{B}^{(0)}$ are three initial matrices, and $\mathbf{L}^{(t)}$ and $\mathbf{U}^{(t)}$ are strictly lower and upper triangular matrices for all $t = 0, 1, 2, \ldots$, respectively. The main advantage of the multiplicative update is that the nonsingularity of $\mathbf{B}$ can be easily maintained through the iterations.

In the same time, the set of matrices is updated as

$$\begin{cases} \mathbf{C}_k^{(t+\frac{1}{2})} = (\mathbf{I} + \mathbf{U}^{(t)})\mathbf{C}_k^{(t)}(\mathbf{I} + \mathbf{U}^{(t)})^H, \\ \mathbf{C}_k^{(t+1)} = (\mathbf{I} + \mathbf{L}^{(t)})\mathbf{C}_k^{(t+\frac{1}{2})}(\mathbf{I} + \mathbf{L}^{(t)})^H, \end{cases}$$

where $\mathbf{C}_k^{(0)} \equiv \mathbf{C}_k$ for all $k = 1, \ldots, K$.

Based on the principle of Jacobi algorithm for finding the eigenvalues of a Hermitian matrix, we consider the updating matrices $\mathbf{L}^{(t)}$ and $\mathbf{U}^{(t)}$ with only one non-zero and unknown element. Hence, the concrete forms of the updating lower triangular matrix $\mathbf{L}^{(t)}$ and upper triangular matrix $\mathbf{U}^{(t)}$ are as follows:

$$\mathbf{L}^{(t)} = \mathbf{L}_{ji}^{(t)} \equiv \begin{pmatrix} 0 & \vdots & 0 & \vdots & 0 \\ \cdots & 0 & \cdots & 0 & \cdots \\ 0 & \vdots & 0 & \vdots & 0 \\ \cdots & l_{ji}^{(t)} & \cdots & 0 & \cdots \\ 0 & \vdots & 0 & \vdots & 0 \end{pmatrix},$$

and

$$\mathbf{U}^{(t)} = \mathbf{U}_{ij}^{(t)} \equiv \begin{pmatrix} 0 & \vdots & 0 & \vdots & 0 \\ \cdots & 0 & \cdots & u_{ij}^{(t)} & \cdots \\ 0 & \vdots & 0 & \vdots & 0 \\ \cdots & 0 & \cdots & 0 & \cdots \\ 0 & \vdots & 0 & \vdots & 0 \end{pmatrix}$$

for all i and j with $j > i$.