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Analytical and numerical solutions for the nonlinear Burgers and advection–diffusion equations by using a semi-analytical iterative method

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ABSTRACT

In this paper, exact solutions have been obtained for 1D, 2D and 3D nonlinear Burgers' equations and systems of equations by implementing an accurate semi-analytical method. This method, originally proposed by Temimi and Ansari and herein named TAM, proved to be efficient and reliable for solving different types of linear and nonlinear problems. This method is characterized by not requiring any restrictive assumptions for the nonlinear terms. The convergence of the method is successfully presented and mathematically proved. In addition, the advection–diffusion equation is also solved by using the TAM to demonstrate the efficiency of this method. Several examples are solved either analytically or numerically, where the accuracy of the numerical solution has been demonstrated by evaluating the absolute and relative errors to show the accuracy of the proposed method. The software used in the current work is Mathematica® 10.

1. Introduction

The Burgers' equation was firstly introduced by Harry Bateman in 1915 [1] and it was subsequently renamed as the Burgers' equation [2]. The Burgers' equation has many applications in science and engineering, especially in problems that have the form of nonlinear equations. The applications of Burgers' equation by mathematical scientists and researchers have become more important and interesting. It has been known that this equation describes different types of phenomena such as modeling of dynamics, heat conduction, acoustic waves, turbulence and many others [2–8]. In most cases, this kind of nonlinear PDE should be solved by using special methods because it does not admit analytical solutions. In recent decades, some scientists and researchers used analytical methods to solve these types of problems such as Adomian decomposition method (ADM), Variational iteration method (VIM), Homotopy perturbation method (HPM), Homotopy analysis method (HAM) and Differential transform method (DTM) [9–13].

Temimi and Ansari have recently proposed a semi-analytical iterative method to solve linear and nonlinear ODEs and PDEs [14,15]. This iterative method has been used recently to achieve exact and approximate solutions for several problems, e.g. solving nonlinear ordinary differential equations [16], solving the Korteweg–de Vries equations [17], solving Duffing equations [18], solving the nonlinear thin film flow problems [19], solving some chemistry problems [20] and solving the Fokker–Planck equations [21].

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The advantages of the TAM are that it does not require any restrictive assumptions for non-linear terms such as the ADM requirement of the so-called Adomian polynomial, avoiding large computational work or using any other parameters as in the VIM. Moreover, it avoided construction of the homotopy and solving the corresponding algebraic equations as in HPM.

The Burgers' equation has various physical interpretations such as modeling of transport phenomena with accumulation, advection, diffusion terms and others. In this sense, to demonstrate the efficiency of the TAM in terms of those interpretations, we dealt with the advection–diffusion equation. This equation is one of the most important problems concerning the interpretation of the phenomena related to the physical interpretations of Burgers equation, some studies can be found in [22–24].

In this paper, the exact solutions for different types of Burgers' equations and systems of Burgers' equations in 1D, 2D and 3D will be achieved by applying the TAM to illustrate the efficiency and accuracy of the proposed method. Moreover, the convergence of the TAM based on the Banach fixed-point theorem will be studied. Furthermore, the advection–diffusion equation will be solved for several examples to demonstrate the reliability of the TAM, with a comparison with other numerical solution.

The present paper has been arranged as follows: in section two the basic idea of the TAM is presented. In section three the convergence of the TAM is discussed. In section four, solving 1D, 2D and 3D forms of the Burgers' equations and resulting systems of equations by the TAM is presented. In section five, the analytical and numerical solutions for some examples of the advection–diffusion equation by using the TAM will be given. Finally, in section six, the conclusion is presented.

2. The basic idea of the TAM

To illustrate the basic concepts of the proposed method, let us consider the following partial differential equation [17]:

$$A(u(x,t)) + g(x,t) = 0, \text{ with the boundary conditions } B(u, \frac{\partial u}{\partial x}) = 0, x \in \Omega. \quad (1)$$

where x is the independent variable, u is the unknown function, g is a known analytical function, A is a general differential operator, B is a boundary operator, $\frac{\partial u}{\partial x}$ denotes a differentiation along the normal drawn outwards from Ω. The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Therefore, Eq. (1) can be rewritten as follows:

$$L(u(x,t)) + N(u(x,t)) + g(x,t) = 0, \quad B(u, \frac{\partial u}{\partial x}) = 0, \quad x \in \Omega. \quad (2)$$

In this method, by the elimination of the nonlinear term, the initial guess is obtained as follows

$$L(u_0(x,t)) + g(x,t) = 0, \quad B(u_0, \frac{\partial u_0}{\partial x}) = 0. \quad (3)$$

To generate the next iteration to the solution, we solve the following problem:

$$L(u_1(x,t)) + g(x,t) + N(u_0(x,t)) = 0, \quad B(u_1, \frac{\partial u_1}{\partial x}) = 0. \quad (4)$$

Then, the $u_{n+1}(x,t)$ iteration can be obtained as

$$L(u_{n+1}(x,t)) + g(x,t) + N(u_n(x,t)) = 0, \quad B(u_{n+1}, \frac{\partial u_{n+1}}{\partial x}) = 0. \quad (5)$$

In fact, each $u_n(x,t)$ is separately a solution of the problem. The iterative method is easy to implement and each solution is better than the previous iteration. Continuing in this manner, a good approximate solution can be obtained with good agreement with the exact solution.

3. Convergence analysis of the TAM

To show the convergence analysis of the TAM, let us begin by introducing the following process for our proposed semi-analytical method. We have the terms in these forms

$$\begin{aligned} v_0 &= u_0(x,t), \\ v_1 &= F[v_0], \\ v_2 &= F[v_0+v_1], \\ &\vdots \\ v_{n+1} &= F[v_0+v_1+\cdots+v_n]. \end{aligned} \quad (6)$$

The operator F can be defined by

$$F[v_k] = S_k - \sum_{i=0}^{k-1} v_i(x,t), \quad k = 1, 2, \ldots. \quad (7)$$

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