ARTICLE IN PRESS
Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

CFS-PML-DEC formulation in two-dimensional convex and non-convex domains

A.S. Moura [a,*], E.J. Silva [b], W.G. Facco [c], R.R. Saldanha [b]

[a] Department of Economy, Federal University of Juiz de Fora, Brazil
[b] Department of Electrical Engineering, Federal University of Minas Gerais, Brazil
[c] General Training Coordination, Federal Institute of Espirito Santo, Brazil

ARTICLE INFO

Article history:
Received 15 September 2017
Received in revised form 12 April 2018
Accepted 15 April 2018
Available online xxxx

Keywords:
Differential forms
Absorbing boundary conditions
Incidence matrix
Convex domains
Non-convex domains
CFS-PML

ABSTRACT

In this paper, the time domain Maxwell's equations are solved using the discrete exterior calculus (DEC) formalism in the two-dimensional space. To truncate the computational domain, the complex frequency-shifted perfectly matched layer (CFS-PML) concept is applied to create a reflectionless artificial layer. The paper presents a new numerical procedure to easily implement the CFS-PML with curved inner boundary. In order to numerically realize the PML, in a simplicial mesh, this paper proposes to utilize the nearest neighbor algorithm to associate point sets to boundary points. The distance from points to the boundary curve defines the attenuation function inside the PML. The performance of the approach is assessed by measuring the reflection error for three numerical experiments.

1. Introduction

In computational electromagnetics [1], curved boundaries are often needed to obtain an appropriate model for wave propagation problems. In numerical simulations, the approach used to impose absorbing boundary conditions considerably affect the accuracy and the computational costs. The perfectly matched layer (PML), originally proposed by *Berenger* [2], requires boundaries aligned with a constant coordinate line, the attenuation direction.

One advantage of the PML over other absorbing boundary conditions is that the PML can be set closer to the radiating sources. Thus, to take full advantage of this feature it is desired to build PML that conforms to the physical boundaries.

These kinds of problems have been investigated as a conformal PML (CPML), some relevant references are the works published by *Teixeira and Chew* [3] *Kuzuoglu and Mittra* [4,5], and *Donderici and Teixeira* [6]. In the latter, a conformal PML is introduced to solve Maxwell's equations in time domain using the mixed finite element formulation (FETD).

A powerful way to address problems of electromagnetic waves is based on the exterior calculus of differential forms [7,8]. This allows us naturally to formulate the mixed FETD method, which is based on Maxwell's first order curl equations. In this case, the unknowns are the electric field circulation along the edges and the magnetic flux through the faces. To approximate the fields, Whitney 1-forms are used to represent the electric field intensity $\mathcal{E}$, and Whitney 2-forms approximate the magnetic flux density $\mathcal{B}$ [9]. When using the leap-frog scheme, the update equations resemble those of finite difference time domain with the exception that the mixed finite element update requires a sparse matrix inversion [10].

Moura et al. [11] present a new formulation to implement the Cartesian CFS-PML for domain truncation in 2-D directly applied to Maxwell's equations written in the language of differential forms. The influence of the CFS-PML parameters on the

* Correspondence to: Department of Economy, Federal University of Juiz de Fora, Av. Dr. Raimundo Rezende, 330 - Centro, Governador Valadares - MG, 35012-140, Brazil.

E-mail addresses: alex.moura@ufjf.edu.br (A.S. Moura), elson@cpdee.ufmg.br (E.J. Silva), werleyfacco@ifes.edu.br (W.G. Facco), rodney@cpdee.ufmg.br (R.R. Saldanha).

https://doi.org/10.1016/j.camwa.2018.04.012

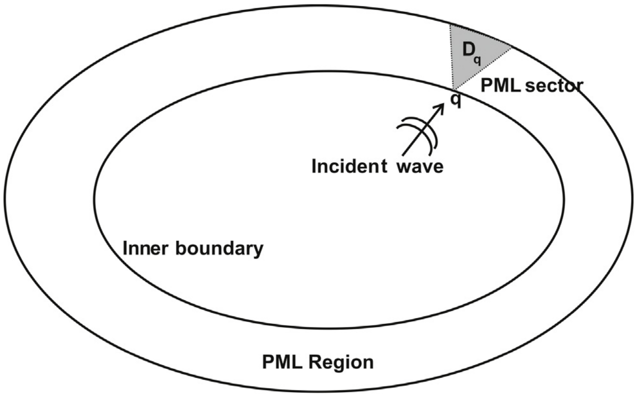

Fig. 1. A PML sector area defined by the nearest neighbors of q.

reflection error is investigated and optimal choices of these parameters are discussed. It is shown that the proposed method is highly absorptive to evanescent modes when computing the wave interaction of elongated structures or sharp corners.

The novelty of this work is a new procedure to build PML with curved inner interfaces. It is based on the nearest neighbor algorithm to select points in the PML region discretized by triangles. It is easily implemented in a code based on the formulation presented in [11].

2. PML realization via nearest neighbors

To explain the rationale behind our approach, we recall that a PML region can be realized by mapping each point (x, y) into a complex point $(\tilde{x}, \tilde{y})$. The imaginary part, which causes wave attenuation, is a function of the shortest distance between the inner boundary and the corresponding PML point. Note that this map can be applied locally and consequently we can obtain a PML of arbitrary shape.

In this paper, we propose to utilize the nearest neighbors algorithm to build PML in the discrete context. Assuming that the layer is discretized by a simplicial mesh each node on the inner surface is a query point, q. We define the set D_q as all points in the PML region that are the nearest neighbors to q. Each point of D_q is complexfied by defining its imaginary part as a function of its Euclidean distance to q.

Fig. 1 presents the geometric idea of the approach, and outgoing waves from the physical domain will attenuate as it propagates in the sector area associated to q.

3. Discrete CFS-PML Maxwell's equations in curved domains

In two-dimensional domains, Maxwell's equations decompose into transverse-electric (TE) and transverse-magnetic (TM) modes. For TE modes, the electric field, $\mathcal{E}$, remains as a 1-form but the magnetic induction, $\mathcal{B}$, becomes a scalar field (the z-component of the vector B), i.e. a 2-form. On the other hand, for TM modes, the electric field is a scalar function, i.e. a 0-form, and $\mathcal{B}$ becomes a 1-form [12]. Inside the PML region, frequency domain Maxwell's equations are written in terms of differential forms as follows [7]:

$$d_s\mathcal{E} = -j\omega\mathcal{B} \tag{1}$$

$$d_s\left(\star_\nu\mathcal{B}\right) = j\omega\star_\epsilon\mathcal{E} \tag{2}$$

where $\star_\epsilon$ and $\star_\nu$ are Hodge star operators associated with the permittivity (ϵ) and permeability $\left(\mu = \nu^{-1}\right)$ of the medium and, in the interior of the PML regions, d_s is the exterior derivative operator modified as

$$d_s = \left(\frac{1}{s_x}\frac{\partial}{\partial x}dx + \frac{1}{s_y}\frac{\partial}{\partial y}dy\right)\wedge . \tag{3}$$

For $p = x, y$ we have $s_p(\omega) = k_p + \frac{\sigma_p}{\alpha_p + j\omega\epsilon_0}$, where σ_p is the conductivity in direction p, α_p and k_p are positive real parameters of which $k_p \geq 1$. In the discretized context, we consider these parameters as being constant within each mesh element.

For the reader unfamiliar with the differential form notation, Eqs. (1)–(3) are rewritten in vector form:

$$\nabla_s \times \bar{\mathbf{E}} = -j\omega\bar{\mathbf{B}} \tag{4}$$

$$\nabla_s \times \left(\nu\bar{\mathbf{B}}\right) = j\omega\epsilon\bar{\mathbf{E}} \tag{5}$$

Please cite this article in press as: A.S. Moura, et al., CFS-PML-DEC formulation in two-dimensional convex and non-convex domains, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.04.012.