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Characteristics of solitary wave, homoclinic breather wave and rogue wave solutions in a (2+1)-dimensional generalized breaking soliton equation ☆

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ABSTRACT

We consider a (2+1)-dimensional generalized breaking soliton (gBS) equation, which describes the interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis. By using Bell's polynomials, we derive a bilinear form of the gBS equation. Based on the resulting Hirota's bilinear equation, we explicitly construct its soliton solutions. Furthermore, by using the extended homoclinic test theory, its homoclinic breather waves and rogue waves are well derived, respectively. It is hoped that our results can enrich the dynamical behavior of the gBS-type equations.

1. Introduction

Nonlinear evolution equations (NLEEs) can describe many nonlinear phenomena in the plasmas media, nonlinear optic substance, finance, etc. There exist many effective methods to find the analytical solutions of NLEEs, including Lie symmetry transformation [1,2], inverse scattering transformation [3], Darboux transformation [4], and Hirota's bilinear theory [5]. Based on these methods, one can try to find many interesting analytical solutions of NLEEs, such as soliton solutions, rational solutions, periodic wave solutions, breather wave solutions, and rogue wave solutions.

Recently, there are many works to find solitary wave solutions for some NLEEs [6–14]. It is worth mentioning that more and more mathematical physicists pay attention to the rogue waves (RWs) in both theoretical predictions and experimental observations. The rogue wave arises in the fields of the deep ocean, Bose–Einstein condensates, nonlinear optic, plasmas, etc. [15–21]. It is well-known that storms and tsunamis caused by typhoon can be predicated hours in advance, but the oceanic RWs suddenly appear from nowhere and disappear without a trace [22]. Peregrine first studied the rational solution of the nonlinear Schrödinger (NLS) equation to explain the rogue waves phenomenon [23]. In addition, there are a lot of works to study the rogue wave solutions of some NLEEs [24–33].

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In this work, we mainly consider the following generalized breaking soliton (gBS) equation [34]:

$$\begin{aligned} u_t + au_{3x} + bu_{2xy} + cuu_x + duu_y + eu_x v = 0, \\ u_y = v_x, \end{aligned} \tag{1.1}$$

where a, b, c, d and e are arbitrary constants. The gBS equation (1.1) describes the interaction of the Riemann wave propagated along the y-axis with a long wave propagated along the x-axis. A class of overturning soliton solutions of the gBS-type equations have been introduced in [35–39]. For $y = x$, by integrating the resulting equation in (1.1), the equation is reduced to the KdV-type equation. Moreover, by taking $u = m_x$, $v = m_y$, $a = c = 0$, $b = 1$, $d = -4$, $e = -2$, Eq. (1.1) reduces to the constant coefficients breaking soliton equation

$$m_{xt} + m_{xxy} - 2m_{xx}m_y - 4m_x m_{xy} = 0, \tag{1.2}$$

which has been studied by Calogero and Degasperis [40]. Particularly, taking $u = m_x$, $v = m_y$, $a = c = 0$, $b = 1$, $d = e = 4$, one can obtain another similar form

$$m_{xt} + m_{xxxy} + 4m_{xx}m_y + 4m_x m_{xy} = 0, \tag{1.3}$$

which has been studied by Bogoyavlenskii. Its overlapping solutions are derived in [41]. In the previous work, some exact solutions including periodic wave solutions, analytical solutions and rational solutions have been constructed in [35–42]. To the best of authors' knowledge, many works for Eq. (1.1) have been done, but the breather waves and rogue waves of Eq. (1.1) have not been studied before.

The primary purpose of this paper is to derive the homoclinic breather wave and rogue wave solutions of Eq. (1.1) by using an effective way. In addition, by using Bell's polynomial, the bilinear equation and soliton solutions of Eq. (1.1) are also obtained.

The structure of this paper is as follows. In Section 2, we mainly introduce Bell's polynomial, and derive the bilinear form of Eq. (1.1). In Section 3, based on Hirota's bilinear method, we can obtain solitons in a natural way. In Section 4, by considering an extended homoclinic test method (EHTM), its homoclinic breather waves and rogue waves are also found. Finally, some conclusions and discussions are provided in Section 5.

2. Bilinear formalism

We introduce a dependent variable transformation

$$u = g(t)q_{xx}, \tag{2.1}$$

where $g = g(t)$ is a function of t. Substituting (2.1) into (1.1), one can obtain the following result:

$$g_t(t)q_{xx} + g(t)q_{xxt} + ag(t)q_{5x} + bg(t)q_{4xy} + cg^2(t)q_{xx}q_{xxx} + dg^2(t)q_{xx}q_{xxy} + eg^2(t)q_{xxx}q_{xy} = 0. \tag{2.2}$$

Integrating above result (2.2) with respect to x, one finds

$$E(q) = q_{xt} + a(q_{4x} + 3q_{xx}^2) + b(3q_{xx}q_{xy} + q_{3xy}) = \sigma, \tag{2.3}$$

under the following constraint conditions:

$$g(t) = 1, c = 6a, d = e = 3b. \tag{2.4}$$

Based on the results in Refs. [43–47], the gBS equation (1.1) can be written in terms of P-polynomial

$$E(q) = P_{xt} + aP_{4x} + bP_{3xy} = \sigma. \tag{2.5}$$

Particularly, taking $\sigma = 0$, Eq. (2.5) reduces to the following equation:

$$E(q) = P_{xt} + aP_{4x} + bP_{3xy} = 0. \tag{2.6}$$

Then, Eq. (2.6) leads to the bilinear equation

$$\left(D_x D_t + aD_x^4 + bD_x^3 D_y\right) F \cdot F = 0, \tag{2.7}$$

under the following transformation:

$$q = 2\ln F \Leftrightarrow u = 2[\ln F]_{xx}. \tag{2.8}$$

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