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Darboux transformation and nonautonomous solitons for a modified Kadomtsev–Petviashvili equation with variable coefficients

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ABSTRACT

We study a modified Kadomtsev–Petviashvili (mKP) equation with variable coefficients, which has important applications in fluid dynamics, ferromagnetics and plasma. The Lax pair, Hirota bilinear form and Darboux transformation (DT) are obtained through the two-singular manifold method. The N-soliton solution with Grammian type in a compact determinant representation is derived from the N-fold DT. A complete classification of the mKP-type solitons is given, namely, the bright and dark solitons, as well as resonant bright and dark solitons. Dynamics of the nonautonomous solitons with parabolic, periodic and kink shapes under different kinds of variable-coefficient oscillations are shown. Interactions between the nonautonomous two bright solitons and two resonant bright solitons are discussed through the asymptotic analysis method.

1. Introduction

It is well known that the modified Kadomtsev–Petviashvili (mKP) equation,

$$u_t - u_{xxx} + 6u^2u_x + 6u_x\partial_x^{-1}u_y - 3\partial_x^{-1}u_{yy} = 0, \qquad (1)$$

is the two-dimensional version of the modified Korteweg–de Vries (mKdV) equation, and the leading approximation to a system which describes the propagations of water waves on the (x, y) plane in fluid dynamics [1], electromagnetic waves in an infinite ferromagnetic thin film [2], ion-acoustic waves in a collisionless plasma [3], and dust ion acoustic waves, dust acoustic waves as well as positron acoustic waves in magnetized plasmas [4–6], to name a few. Eq. (1) has been introduced in many different approaches under various mathematical or physical backgrounds, such as the inverse scattering transform [7], the nonlinearization of Lax pairs technique [8], and the reductive perturbation method [4]. Particularly, Saha and Chatterjee [4] derived the mKP equation from dust ion acoustic waves in a magnetized dusty plasma with q-nonextensive velocity distributed electrons by discussing higher order coefficients of the small parameter ϵ, which reveals the important applications of Eq. (1) to describe the nonlinear wave features in magnetosphere of the Earth. Thus far, the completely integrability and explicit solutions such as the soliton solution, algebraic geometry solution, periodic traveling wave solution, kink and anti-kink wave solutions for Eq. (1) have been receiving quite a lot of attention in recent decades [4–11].

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With regard to explicit solutions of the integrable equations, Darboux transformation (DT) is one of the most effective tools to solve this problem. In general, DT can be constructed through the gauge transformation [12–20], the Riemann–Hilbert approach [21] and the Painlevé analysis [9], etc. Among them, the singular or two-singular manifold method from Painlevé analysis proposed by Estévez opens a new way to construct not only the Lax pair, Hirota τ-function but also the DT for the integrable equations [9]. Very recently, Estévez has employed the singular manifold method to derive the Lax pair, DT and lump solutions for a higher-order nonlinear equation in $2+1$ dimension [22], and Li has used the two-singular manifold method to obtain the Lax pair, binary DT and nonautonomous soliton solutions for a variable-coefficient nonisospectral mKP equation [23].

In this paper, we study a general mKP equation with variable coefficients,

$$\begin{aligned} u_t &= b(t)(u_{xxx}-6u^2u_x)+k_1(t)(xu_x+u)+s_1(t)u_x+6b(t)[\lambda p_1(t)^2+f_0(t)+xf(t) \\ &\quad -g(t)\partial_x^{-1}u_y]u_x+3b(t)g(t)^2\partial_x^{-1}u_{yy}+[k_2(t)y+s_2(t)]u_y+6b(t)f(t)u, \end{aligned} \tag{2}$$

where $u = u(x, y, t)$ is the amplitude or elevation of the relevant wave, (x, y) and t stand for the spacial and temporal coordinates, respectively. The coefficients $b(t), g(t), p_1(t), f_0(t), f(t), k_i(t), s_i(t), (i = 1, 2)$ are arbitrary functions of the variable t, and can play an important role to simulate the propagation of nonlinear waves in inhomogeneous media when considering the nonuniformities of boundaries and effects of external forces [24,25]. Eq. (2) was first proposed by Zhu to study the Miura transformation corresponding to a general KP equation with variable coefficients [26] (see Eq. (14) in the reference), and can be reduced to the standard mKP equation [1], cylindrical mKP equation [27], and extended mKP equation with multi-front waves [28] when choosing the adequate variable-coefficient functions, respectively. The infinite conservation laws [26], explicit solutions with complex coefficients [29], and soliton-like solutions beyond the traveling waves [30] have been obtained. Nevertheless, to our knowledge, the integrability such as the Lax pair and Hirota bilinear form [31–37], the construction of DT, and the general N-soliton solution for Eq. (2) have not been reported elsewhere.

In this work, we focus on the DT and nonautonomous solitons for Eq. (2) by virtue of the two-singular manifold method. The N-soliton solution with Grammian type in a compact $N \times N$ determinant is derived, and a complete classification for the mKP-type solitons is given. Dynamics of the nonautonomous solitons under different types of variable-coefficient oscillations, and interactions between the nonautonomous two bright solitons and two resonant bright solitons are discussed. The present paper is organized as follows. In Section 2, the Lax pair and DT of Eq. (2) are derived. In Section 3, the N-soliton solution is obtained, the dynamics of the nonautonomous one- and two-soliton solutions are shown, and the interactions between the nonautonomous two solitons are discussed. In Section 4, we give the conclusions.

2. Lax pair and Darboux transformation

In this section, we utilize the two-singular manifold method [9] to derive the Lax pair and DT for Eq. (2). For this end, we first rewrite Eq. (2) as follows

$$\begin{aligned} u_t &= b(t)(u_{xxx}-6u^2u_x)+k_1(t)(xu_x+u)+s_1(t)u_x+6b(t)[\lambda p_1(t)^2+f_0(t)+xf(t) \\ &\quad -g(t)\omega]u_x+3b(t)g(t)^2\omega_y+[k_2(t)y+s_2(t)]\omega_x+6b(t)f(t)u, \end{aligned} \tag{3a}$$

$$u_y = \omega_x. \tag{3b}$$

Then, we consider the following two truncated Painlevé expansions

$$u' = u + \left(\frac{\phi_x}{\phi} - \frac{\sigma_x}{\sigma}\right), \tag{4a}$$

$$\omega' = \omega + \left(\frac{\phi_y}{\phi} - \frac{\sigma_y}{\sigma}\right), \tag{4b}$$

where ϕ is the singular manifold for $\epsilon = 1$ and σ is that for $\epsilon = -1$. Hereby, if we assume

$$\frac{\phi_x}{\phi}\frac{\sigma_x}{\sigma} = A\frac{\phi_x}{\phi} + B\frac{\sigma_x}{\sigma}, \tag{5}$$

then inserting Eqs. (4) and (5) into Eq. (3) provides the list of equations

$$A = \frac{1}{2}[2u + v_1 + g(t)\tau_1],\ B = -\frac{1}{2}[2u - v_2 + g(t)\tau_2],$$

and

$$\begin{aligned} &b(t)v_1^2+3b(t)g(t)^2\tau_1^2-w_1+4b(t)v_{1x}+[k_2(t)y+s_2(t)]\tau_1-6b(t)u^2+k_1(t)x+6b(t)\lambda p_1(t)^2 \\ &\quad +6b(t)f_0(t)-6b(t)g(t)\omega+6b(t)u_x+s_1(t)+6b(t)f(t)x=0, \\ &b(t)v_2^2+3b(t)g(t)^2\tau_2^2-w_2+4b(t)v_{2x}+[k_2(t)y+s_2(t)]\tau_2-6b(t)u^2+k_1(t)x+6b(t)\lambda p_1(t)^2 \\ &\quad +6b(t)f_0(t)-6b(t)g(t)\omega-6b(t)u_x+s_1(t)+6b(t)f(t)x=0, \end{aligned}$$