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Parameter limit method and its application in the (4+1)-dimensional Fokas equation

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ABSTRACT

A new Hirota's bilinear form of the (4+1)-dimensional Fokas equation is obtained by an appropriate wave transformation. Applying the parameter limit method introduced in this paper, we obtain two homoclinic solutions and a rogue wave solution of (4+1)-dimensional Fokas equation. Besides, we also discuss the interaction between rogue wave solution and different forms of stripe solitons (one stripe soliton and double stripe solitons), and investigate the annihilation and emergence of rogue waves. Finally, the parametric reasons of soliton fusion and fission are discussed, furthermore, the fusion and fission phenomena of soliton are simulated by three-dimensional plots.

1. Introduction

It is well known that much physical phenomena can be obtained or proved by the solitary wave solution of nonlinear partial differential equations (NLPDEs). Therefore, the investigation and solution of solitary waves for NLPDEs has become more and more attractive. Many methods have been proposed over these years for finding solitary wave solutions, such as the Darboux transformation method [1], Lie group method [2], inverse scattering method [3], multiple exp-function method [4,5], three-wave method [6] and extended homoclinic test method [7]. Rogue wave is also known as killer wave, giant wave, freak wave, monster wave, extreme wave, or abnormal wave [8], as a special type of solitary wave, it always has two or more times amplitude higher than its surrounding waves and generally forms in a short time for which people think that it comes from nowhere. Recently, rogue wave solution attracts a lot of attention. Ma and his collaborators [9,10] have applied a new direct method based on quadratic function to obtain the lump solutions, which have the characteristics of rogue wave solutions, so the lump solutions of this form are also called rogue wave solutions by some scholars [11,12]. Meanwhile, rogue wave solutions and interaction between rogue waves and other forms of solitary waves were presented for many nonlinear systems [13–19].

Compared with the lower-dimensional NLPDEs, more attention is paid to the study of high-dimensional systems, which can provide more useful information and lead to further applications. In this paper, we consider the $(4+1)$-dimensional Fokas equation

$$u_{xt} - \frac{1}{4}u_{xxxy} + \frac{1}{4}u_{xyyy} + \frac{3}{2}(u^2)_{xy} - \frac{3}{2}u_{zw} = 0, \tag{1}$$

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where $u : R_x \times R_y \times R_z \times R_w \times R_t \to R$. Eq. (1) comes from the process of extending the integrable Kadomtsev–Petviashvili and Davey–Stewartson equations to some higher-dimensional nonlinear wave equations by Fokas [20]. Due to important applications of higher-dimensional equations in real world problems, it is necessary to investigate its analytic solutions. Therefore, the (4 + 1)-dimensional Fokas equation is studied by many authors. Recently, Al-Amr et al. reported its some traveling wave solutions via using two distinct methods [21]. Zhang et al. acquired its some multi-soliton solutions (single-soliton solution, double-soliton solution and three-soliton solution) by using the Hirota bilinear method [22]. He studied its traveling wave solutions by using extended F-Expansion method and its variant [23]. Lee et al. discussed its exact solutions by using modified tanh-coth method, the exp-function method and extended Jacobi elliptic function method [24]. Yang et al. obtained its Jacobi elliptic double periodic solutions, rational solutions and hyperbolic function solutions by investigating its symmetries [25]. Kim et al. obtained its hyperbolic function solutions, trigonometric function solutions and rational solution by applying G/G'-expansion method [26], respectively.

However, the rogue wave solutions and interaction between rogue waves and other formal solitons of (4 + 1)-dimensional Fokas equation have not been presented in the previous work. Here, the outline of our article is as follows: In Section 2, the parameter limit method and its application in interaction is described in detail. In Section 3, an exact rogue wave solution and two homoclinic solutions are obtained by application parameter limit method. In Section 4, we study the interaction between one stripe soliton (and double stripe solitons) and rogue waves, the fusion and fission phenomena of solitons are investigated and simulated by three-dimensional plots, and we also discuss the parametric reasons of soliton fusion and fission. Finally, the conclusion is given in Section 5.

2. Parameter limit method

Consider any NLPDEs as the following form

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, \ldots) = 0, \tag{2}$$

where $u = u(x, y, t)$, F is a polynomial of u and its derivatives. The essence of parameter limit method can be presented in the following steps.

Step 1: By Painlevé analysis, we make a transformation as

$$u = T(f), \tag{3}$$

where $f(x, y, t)$ is new unknown function.

Step 2: The bilinear equation of Eq. (2) can obtained by substituting Eq. (3) into Eq. (2),

$$G(D_t, D_x, D_y, D_z; f, f) = 0, \tag{4}$$

where the bilinear operator D is defined by [27]

$$D_x^m D_y^k f \cdot g = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^k f(x, y, t) \cdot g(x', y', t')|_{(x,y,t)=(x',y',t')}. \tag{5}$$

Step 3: Select the appropriate test function, such as

$$f(x, y, t) = 1 + m_1(e^{ip_1\theta} + e^{-ip_1\theta})e^{p_2\vartheta} + m_2 e^{2(p_2\vartheta)} \tag{6}$$

or

$$f(x, y, t) = e^{-p_1\theta} + m_2 \cos(p_2\vartheta) + m_1 e^{p_1\theta}, \tag{7}$$

where $\theta = x + a_1 y + b_1 t + c_1$, $\vartheta = x + a_2 y + b_2 t + c_2$ or their deformation, and $a_i, b_i, c_i, p_i, m_i (i = 1, 2)$ are some arbitrary constants.

Step 4: Substituting Eq. (6) (or Eq. (7)) into bilinear Eq. (4), and collecting all coefficients of $e^{-p_1\theta}$, $e^{p_1\theta}$, $\cos(p_2\vartheta)$, $\sin(p_2\vartheta)$ and constant term to zero, then, some homoclinic solutions are obtained.

Step 5: When $m_2 < 0$, taking $m_1 = 1$ in the homoclinic solution obtained from the step 4, and finding functional relationships between p_2 and p_1, which needs to satisfy the conditions: $\lim_{p_1\to 0} p_2 = \lim_{p_1\to 0} h(p_1) = 0$, here p_2 is a function of p_1, such as $p_2 = kp_1$. Then, a rogue wave solution about θ and ϑ is obtained by letting $p_1 \to 0$.

Step 6: In order to study interaction between rogue waves and other formal solitons, on the basis of step 3, we choose a new test function

$$f(x, y, t) = a_0 + \theta^2 + \vartheta^2 + g(x, y, t), \tag{8}$$

where $g(x, y, t)$ is an unknown function that will be chosen, such as $g(x, y, t) = \delta e^{(a_3 x + b_3 y + c_3 t + d_3)}$ or $g(x, y, t) = \delta \cosh(a_3 x + b_3 y + c_3 t + d_3)$.

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