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Well-posedness and exponential stability for coupled Lamé system with viscoelastic term and strong damping

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ABSTRACT

In this paper, we consider a coupled Lamé system with a viscoelastic term and a strong damping. We prove well posedness by using Faedo–Galerkin method and establish an exponential decay result by introducing a suitable Lyapunov functional.

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^3$ with smooth boundary $\partial\Omega$. Let us consider the following coupled Lamé system:

$$\begin{cases} u_{tt}(x,t) + \alpha v - \Delta_e u(x,t) + \displaystyle\int_0^t g_1(t-s)\Delta u(x,s)ds - \mu_1 \Delta u_t(x,t) = 0, & \text{in } \Omega \times (0,+\infty), \\ v_{tt}(x,t) + \alpha u - \Delta_e v(x,t) + \displaystyle\int_0^t g_2(t-s)\Delta v(x,s)ds - \mu_2 \Delta v_t(x,t) = 0, & \text{in } \Omega \times (0,+\infty), \\ u(x,t) = v(x,t) = 0 & \text{on } \partial\Omega \times (0,+\infty), \\ (u(x,0), v(x,0)) = (u_0(x), v_0(x)) & \text{in } \Omega, \\ (u_t(x,0), v_t(x,0)) = (u_1(x), v_1(x)) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where μ_1, μ_2 are positive constants and (u_0, u_1, v_0, v_1) are given history and initial data. Here Δ denotes the Laplacian operator and Δ_e denotes the elasticity operator, which is the 3×3 matrix-valued differential operator defined by

$$\Delta_e u = \mu \Delta u + (\lambda + \mu)\nabla(\text{div u}), \quad u = (u_1, u_2, u_3)^T$$

and μ and λ are the Lamé constants which satisfy the conditions

$$\mu > 0, \quad \lambda + \mu \geq 0. \tag{1.2}$$

The problem of stabilization of coupled systems has also been studied by several authors see [1–7] and the references therein. Under certain conditions imposed on the subset where the damping term is effective, Komornik [7] proves uniform stabilization of the solutions of a pair of hyperbolic systems coupled in velocities. Alabau et al. [4] studied the indirect internal

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stabilization of weakly coupled systems where the damping is effective in the whole domain. They prove that the behavior of the first equation is sufficient to stabilize the total system and to have polynomial decay for sufficiently smooth solutions.

For coupled systems in thermoelasticity, R. Racke [5] considered the following system:

$$\begin{cases} u_{tt}(x,t) - au_{xx}(x,t-\tau) + b\theta_x(x,t) = 0, & \text{in } (0,L)\times(0,\infty), \\ \theta_t(x,t) - d\theta_{xx}(x,t) + bu_{tx}(x,t) = 0, & \text{in } (0,L)\times(0,\infty). \end{cases}$$

He proved that the internal time delay leads to ill-posedness of the system. However, the system without delay is exponentially stable.

In [8], Bchatnia and Daoulatli studied behavior of the energy for solutions to a Lamé system on a bounded domain with localized nonlinear damping and external force, that is

$$\begin{cases} u''(x,t) - \Delta_e u(x,t) + a(x)g(u'(x,t)) = f(t,x) & \text{in } \Omega\times\mathbb{R}^+ \\ u(x,t) = 0 & \text{on } \partial\Omega\times\mathbb{R}^+ \\ u(x,0) = u_0(x) \quad u'(x,0) = u_1(x) & \text{on } \Omega. \end{cases} \tag{1.3}$$

Recently, Guesmia and Bchatnia [9] studied behavior of the energy for solutions to a Lamé system with infinite memories in a bounded domain, that is

$$\begin{cases} u''(x,t) - \Delta_e u(x,t) + \displaystyle\int_0^{+\infty} g(s)\Delta u(t-s)\,ds = 0 & \text{in } \Omega\times\mathbb{R}^+ \\ u(x,t) = 0 & \text{on } \partial\Omega\times\mathbb{R}^+ \\ u(x,-t) = u_0(x) & \text{in } \Omega\times\mathbb{R}^+ \\ u(x,0) = u_0(x) \quad u'(x,0) = u_1(x) & \text{on } \Omega. \end{cases} \tag{1.4}$$

Very recently, in [2], Beniani et al. considered the following Lamé system with time varying delay term:

$$\begin{cases} u''(x,t) - \Delta_e u(x,t) + \mu_1 g_1(u'(x,t)) + \mu_2 g_2(u'(x,t-\tau(t))) = 0 & \text{in } \Omega\times\mathbb{R}^+ \\ u(x,t) = 0 & \text{on } \partial\Omega\times\mathbb{R}^+ \\ u'(x,t-\tau(0)) = f_0(x,t-\tau(0)) \text{ in } \partial\Omega\times(0,\tau(0)) & \\ u(x,0) = u_0(x) \quad u'(x,0) = u_1(x) \text{ on } \Omega & \end{cases} \tag{1.5}$$

and under suitable conditions, they proved general decay of energy.

In [10], authors considered the following problem:

$$\begin{cases} u_{tt} - \Delta_x u(x,t) - \mu_1\Delta u_t(x,t) - \displaystyle\int_{\tau_1}^{\tau_2} \mu_2(s)\Delta u_t(x,t-s)ds = 0 \text{ in } \Omega\times(0,+\infty), \\ u = 0 \text{ on } \Gamma\times[0,+\infty), \\ u(x,0) = u_0(x) \quad u'(x,0) = u_1(x) \text{ on } \Omega, \\ u_t(x,-t) = f_0(x,-t) \ \ 0 < t \le \tau_2 \end{cases} \tag{1.6}$$

and under the assumption

$$\mu_1 > \int_{\tau_1}^{\tau_2} |\mu_2| ds \tag{1.7}$$

they proved that the solution is exponentially stable.

Recently, Ferhat and Hakem [11] considered the weak viscoelastic wave equation in bounded domain with dynamic boundary conditions, and nonlinear delay term:

$$\begin{cases} u_{tt} - \Delta_x u(x,t) - \delta\Delta u_t(x,t) - \sigma(t)\displaystyle\int_0^t g(t-s)\Delta u(x,s)\,ds = 0 \text{ in } \Omega\times(0,+\infty), \\ u = 0 \text{ on } \Gamma_0\times(0,+\infty), \\ u_{tt} + a\Big[\dfrac{\partial u}{\partial\nu}(x,t) + \delta\dfrac{\partial u_t}{\partial\nu}(x,t) - \sigma(t)\displaystyle\int_0^t g(t-s)\Delta u(x,s)\dfrac{\partial u}{\partial\nu}(x,s)\,ds \\ \quad + \mu_1|u_t(x,t)|^{m-1}u_t(x,t) + \mu_2|u_t(x,t-\tau)|^{m-1}u_t(x,t-\tau)\Big] = 0 \text{ on } \Gamma_1\times(0,+\infty) \\ u(x,0) = u_0(x) \quad u'(x,0) = u_1(x) \text{ on } \Omega, \\ u_t(x,t-\tau) = f_0(x,t-\tau) \text{ on } \Gamma_1\times(0,+\infty). \end{cases} \tag{1.8}$$

Under suitable conditions on the initial data and the relaxation function, they proved general decay of energy.

The paper is organized as follows. In Section 2, we give some materials needed for our work and state our main results. The well-posedness of the problem is analyzed in Section 3, by using Faedo–Galerkin method. In Section 4, we prove the exponential decay of the energy when time goes to infinity.