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Spatio-temporal dynamics near the steady state of a planktonic system[☆]

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ABSTRACT

The study of spatio-temporal behaviour of ecological systems is fundamentally important as it can provide deep understanding of species interaction and predict the effects of environmental changes. In this paper, we first propose a spatial model with prey taxis for planktonic systems, in which we also consider the herb behaviour in prey and effect of the hyperbolic mortality rate. Applying the homogeneous Neumann boundary condition to the model and using prey-tactic sensitivity coefficient as bifurcation parameter, we then detailedly analyse the stability and bifurcation of the steady state of the system: firstly, we carry out a study of the equilibrium bifurcation, showing the occurrence of fold bifurcation, Hopf bifurcation and the BT bifurcation; then by using an abstract bifurcation theory and taking prey-tactic sensitivity coefficient as the bifurcation parameter, we investigate the Turing–Hopf bifurcation, obtaining a branch of stable non-constant solutions bifurcating from the positive equilibrium, and our results show that prey-taxis can yield the occurrence of spatio-temporal patterns; finally, numerical simulations are carried out to illustrate our theoretical results, showing the existence of a periodic solution when the prey-tactic sensitivity coefficient is away from the critical value.

1. Introduction

In order to better understand interactions between different species in ecological systems, mathematical models are developed, such as the well-known predator–prey model and its variations. Zhang and Ma et al. [1] proposed a prey–predator model with nonlinear state feedback control, Meng [2] proposed a predator–prey model with selective disturbance, and authors of [3] proposed a stochastic non-autonomous predator–prey system with impulsive effects, to name but a few. In the formulation of these models, the type of functional response and the form of the mortality terms play an important role, such as when formulating plankton population models, a linear mortality is used if the biomass of the predator does not fluctuate, quadratic mortality if biomass of the predator is proportional to that of the zooplankton [4–6]. Braza [7] proposed

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the following model with square root functional response and herd behaviour in prey

$$\frac{dx}{dt} = x(1-x) - \frac{\sqrt{x}y}{1+\alpha\sqrt{x}},$$
$$\frac{dy}{dt} = -sy + \frac{c\sqrt{x}y}{1+\alpha\sqrt{x}},$$

which was then generalised to the one with diffusion in [8–10], where due to the difficulty in analysis, authors considered the case of $\alpha = 0$. Yuan et al. [8] showed that the system had a unique positive equilibrium, at which they investigated the formation of spatial patterns by using amplitude equations and found two types of patterns and also mixture of these two patterns. Using the death rate of the predator as the control parameter, they also investigated the pattern selection, suggesting the large death rate corresponds to the hexagonal pattern, while small death rate for stripes. Xu and Song [9] considered a case with herd behaviour and quadratic mortality and obtained the corresponding bifurcation diagrams showing the relative diffusion rate is a key point for the occurrence of Turing instability. Reference [10] considered the spatiotemporal patterns induced by cross-diffusion, which may result in "labyrinth" patterns or "black-eye" patterns. Duque and Lizan [11] proposed a predator–prey model with special mortality $Q(v) = \frac{\gamma_1+\delta v}{1+v}$ and investigated its global dynamics, showing the system is dissipative and permanent. Aly et al. [12] also studied a such model with mortality $Q(v)$ and observed small amplitude patterns by using amplitude equations. To consider the effects of diffusion on the spatial dynamics, Zhang et al. [13] proposed a predator–prey model with hyperbolic mortality and observed the formation of some elementary two-dimensional patterns. As pointed above or in references [4–6], when modelling planktonic systems, if no data are available for predation rates, linear mortality can be assumed as the simplest and reasonable way; if predator is cannibalistic or some predator population is proportional to its prey, then quadratic mortality can be employed; finally, for defined predator and fairly detailed data are available, hyperbolic mortality would be more reasonable due to the properties of the hyperbolic function compared with that of the linear and quadratic functions, particularly if the predator is assumed satiable or the concentration of zooplankton varies.

To investigate the effects of prey-taxis on spatial dynamics model, Ainseba and his colleagues [14] investigated the existence of weak solutions by using Schauder fixed-point theorem, the uniqueness of solution via duality technique and the linear stability around equilibrium for the following model

$$\begin{cases} \frac{\partial u}{\partial t} - d_1 \Delta u = k(u) - \pi(u)v \quad \text{in} \quad \Omega \times (0,T), \\ \frac{\partial v}{\partial t} - d_2 \Delta v + \nabla\cdot(\chi\phi(v)v\nabla u) = -av + e\pi(u)v \quad \text{in} \quad \Omega\times(0,T), \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) > 0, \quad v(x,0) = v_0(x) > 0 \quad \text{in} \quad \Omega, \end{cases} \tag{1.1}$$

where Ω represents a bounded domain in $\mathbb{R}^n$ ($n \geq 1$ is a positive integer) with a smooth boundary $\partial\Omega$. ν denotes the outward unit normal vector of $\partial\Omega$, d_1 and d_2 are the diffusion constants. χ denotes the prey-tactic sensitivity coefficient, $\chi\phi(v)$ is the prey-tactic sensitivity, the term $\chi\phi(v)v\nabla u$ denotes the velocity of predators moving up the gradient of the prey with

$$\phi(v) = \begin{cases} 1 - \frac{v}{v_m}, & v \leq v_m, \\ 0, & v > v_m. \end{cases}$$

Here, threshold v_m is the maximal density of predators such that $\phi(v_m) = 0$ and biologically $\phi(v)$ is a repulsive switch implying that the predators stop accumulating to attract the preys [14]. By the contraction mapping principle, together with L^p estimates and Schauder estimates of parabolic equations, Tao [15] considered the global existence and uniqueness of classical solutions to system (1.1). As the continuous work of papers [14,15], the existence and stability of steady-state solutions to model (1.1) have been studied in [16] and obtained that the prey-tactic sensitivity coefficient delays the stability of the unique positive constant solution. Furthermore, He and Zheng [17] proved that the global classical solutions are globally bounded. The global stability of the positive constant equilibrium of diffusion systems has also been studied in [18–21]. Yousefnezhad and Mohammadi [18] studied a diffusive predator–prey model with general functional responses and prey-taxis, where the functional response satisfies the Lipschitz condition.

Recently, the predator–prey models with hyperbolic mortality have attracted great attention of researchers, for example [13,22–26]. Zhang and his colleagues [13] investigated the effects of diffusion on the spatial dynamics of a predator–prey model with hyperbolic mortality in predator population and Holling type II functional response and without prey-taxis; Sambath et al. [22] considered the Hopf bifurcation of a reaction–diffusion predator–prey model with hyperbolic mortality and Holling type II response; and by incorporating prey refuge and time delay into the model of [22], Yang and Zhang [23] investigated the Hopf bifurcation properties.

When modelling a planktonic system, Edwards and Yool [5] suggested the hyperbolic mortality to reflect the fact that the population of zooplankton varies, motivated by which and the aforementioned works, in this paper we propose a planktonic