ARTICLE IN PRESS

Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Rational solutions in Grammian form for the (3 + 1)-dimensional generalized shallow water wave equation

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ARTICLE INFO

Article history:
Received 5 February 2018
Received in revised form 21 March 2018
Accepted 30 March 2018
Available online xxxx

Keywords:
Kadomtsev–Petviashvili hierarchy reduction
(3 + 1)-dimensional generalized shallow water wave equation
Rational solutions
Lump soliton

ABSTRACT

In this paper, the (3 + 1)-dimensional generalized shallow water wave equation is investigated using the Hirota bilinear method and Kadomtsev–Petviashvili hierarchy reduction. The explicit rational solutions for such an equation have been presented in the Grammian form. Based on the Grammian form solution for the equation, the one-rational, two-rational and three-order rational solutions are obtained. When complex parameters p_i with nonzero real and imaginary parts are chosen, the lump soliton solutions which are localized in all directions for the (3 + 1)-dimensional generalized shallow water wave equation can be derived from the corresponding rational solutions. As the figures illustrate, the one-lump soliton solution with one peak and one trough propagates stably on the (x, y) plane. The two-lump solitons with different velocities interact with each other and separate with their original shapes and propagation directions. Different from the case of two-lump solitons, the propagation directions of the third-order lump solitons change after the interaction.

1. Introduction

In soliton theory, the explicit solutions play very important role in understanding the wave propagation characteristics and dynamic features that the nonlinear evolution equations can describe in various fields, so that searching for the explicit solutions is always one of the key research subjects [1–6]. Recently, rational solutions [7–16] have been attracting particularly attention of scholars. Several methods have been developed to derive the explicit solutions in rational form of the nonlinear evolution equations, such as the Hirota bilinear method, Darboux transformation and the Kadomtsev–Petviashvili (KP) hierarchy reduction and so on [17–24]. Lump soliton solution is one special kind of rational solutions, which is localized in all directions in the space [25–28]. And it is said to describe nonlinear patterns in the plasma, optic medium and shallow water with the dominated surface tension [29–31]. By virtue of the quadratic function method [32–34], the lump soliton solutions and interaction solutions between lumps and other kinds of solutions have been given for several equations in the previous works, among which the propagation of lump solitons in the (3 + 1)-dimensional nonlinear evolution equations has been paid much attention [35–37]. For understanding the interaction process of the lump solitons, the multiple or higher-order lump soliton solutions are worthy to study.

In this paper, the investigation is on the (3 + 1)-dimensional generalized shallow water wave (SWW) equation

$$u_{xxxy} - 3\,u_x u_{xy} - 3\,u_y u_{xx} + u_{yt} - u_{xz} = 0, \tag{1}$$

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https://doi.org/10.1016/j.camwa.2018.03.046

which can be used in tidal waves, river and irrigation flows, tsunami prediction as well as the weather simulations, and have been researched in kinds of ways. The authors obtained the periodic solitary wave solutions of Eq. (1) using the auto-Bäcklund transformation and a direct test function [38]. The soliton-type solutions of Eq. (1) were derived by the generalized tanh algorithm method [39], and the traveling wave solutions were given using the (G'/G) expansion method [40]. Tang [41] et al. studied the equation utilizing the Pfaffian.

In this paper, by means of the Hirota bilinear method and KP hierarchy reduction, the rational solution for Eq. (1) in terms of Gramm determinant will be constructed. Through proper parameters choice, different pattern of rational solutions will be given, including the multiple and higher-order lump solitons. Based on the explicit expressions of the solution, the propagation features for different type of rational solutions with proper parameters will be discussed.

2. Rational solutions in Grammian form

Under the dependent variable transformation

$$u = -2(\ln f)_x, \tag{2}$$

Eq. (1) can be transformed into the following bilinear form

$$\left(D_x^3 D_y + D_y D_t - D_x D_z\right) f \cdot f = 0, \tag{3}$$

with f being a real function, D_x, D_y, D_z and D_t are the bilinear differential operators defined by [1]

$$D_x^{n_1} D_y^{n_2} D_z^{n_3} D_t^{n_4}(G \cdot F) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^{n_1} \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^{n_2} \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^{n_3} \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^{n_4} G(x, y, z, t) F(x', y', z', t')|_{x'=x, y'=y, z'=z, t'=t},$$

with n_1, n_2, n_3 and n_4 as the non-negative integers.

In the following, the rational solutions of Eq. (1) in the Grammian form will be presented. Using the KP hierarchy reduction, Eq. (3) has the solution,

$$f = \tau_0, \qquad \tau_0 = \det_{1 \le i,j \le N}(m_{i,j}), \tag{4}$$

with the matrix elements given by

$$m_{i,j} = \sum_{k=0}^{n_i} c_{ik} \left(p_i \frac{\partial}{\partial p_i} + \xi_i'\right)^{n_i - k} \sum_{l=0}^{n_j} c_{jl}^* \left(p_j^* \frac{\partial}{\partial p_j^*} + \xi_j'^*\right)^{n_j - l} \frac{1}{p_i + p_j^*}, \tag{5}$$

with

$$\xi_i' = p_i x + 2\,i\,p_i^2 y + 6 p_i^3 t + 12\,i\,p_i^4 z, \tag{6}$$

where p_i, p_j and c_{ik}, c_{jl} are the complex constants, i, j, n_i and n_j are all positive integers, and "$*$" means the complex conjugation. We can normalize $c_{j0} = 1$ by a scaling of f, and hereafter set $c_{j0} = 1$ without loss of generality in Section 3.

Actually, when functions $m_{i,j}^{(n)}$, $\varphi_i^{(n)}$ and $\psi_j^{(n)}$ satisfy the differential and difference relations,

$$\partial_{x_1} m_{i,j}^{(n)} = \varphi_i^{(n)} \psi_j^{(n)}, \qquad \partial_{x_2} m_{i,j}^{(n)} = (\partial_{x_1} \varphi_i^{(n)}) \psi_j^{(n)} - \varphi_i^{(n)} (\partial_{x_1} \psi_j^{(n)}), \tag{7}$$

$$\partial_{x_3} m_{i,j}^{(n)} = (\partial_{x_1}^2 \varphi_i^{(n)}) \psi_j^{(n)} + \varphi_i^{(n)} (\partial_{x_1}^2 \psi_j^{(n)}) - (\partial_{x_1} \varphi_i^{(n)})(\partial_{x_1} \psi_j^{(n)}), \tag{8}$$

$$\partial_{x_4} m_{i,j}^{(n)} = (\partial_{x_1}^3 \varphi_i^{(n)}) \psi_j^{(n)} - (\partial_{x_1}^2 \varphi_i^{(n)})(\partial_{x_1} \psi_j^{(n)}) + (\partial_{x_1} \varphi_i^{(n)})(\partial_{x_1}^2 \psi_j^{(n)}) - \varphi_i^{(n)} (\partial_{x_1}^3 \psi_j^{(n)}), \tag{9}$$

$$m_{i,j}^{(n+1)} = m_{i,j}^{(n)} + \varphi_i^{(n)} \psi_j^{(n+1)}, \quad \partial_{x_k} \varphi_i^{(n)} = \varphi_i^{(n+k)}, \quad \partial_{x_k} \psi_j^{(n)} = -\psi_j^{(n-k)}, \quad k = 1, 2, 3, 4. \tag{10}$$

with $\varphi_i^{(n)}$ and $\psi_j^{(n)}$ as functions of x_k $(k = 1, 2, 3, 4)$, the determinant

$$\tau_n = \det_{1 \le i,j \le N}(m_{i,j}^{(n)}) \tag{11}$$

satisfies the bilinear equation [1]

$$(D_{x_1}^3 D_{x_2} + 2 D_{x_2} D_{x_3} - 3 D_{x_1} D_{x_4}) \tau_n \cdot \tau_n = 0. \tag{12}$$

In order to obtain rational solutions of the $(3+1)$-dimensional generalized SWW equation (1), the following functions satisfying the above relations are selected,

$$\hat{\varphi}_i^{(n)} = A_i\, p_i^n e^{\hat{\xi}_i}, \quad \hat{\psi}_j^{(n)} = B_j (-q_j)^{-n} e^{\hat{\eta}_j}, \tag{13}$$

$$\hat{m}_{i,j}^{(n)} = A_i B_j \frac{1}{p_i + q_j} \left(-\frac{p_i}{q_j}\right)^n e^{\hat{\xi}_i + \hat{\eta}_j}\ , \tag{14}$$