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Collision of characteristic shock with weak discontinuity in non-ideal magnetogasdynamics

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ABSTRACT

In this paper, we consider a quasilinear hyperbolic system of partial differential equations (PDEs) governing unsteady planar or radially symmetric motion of an inviscid, perfectly conducting and non-ideal gas in which the effect of magnetic field is significant. A particular exact solution to the governing system, which exhibits space–time dependence, is derived using Lie group symmetry analysis. The evolutionary behavior of a weak discontinuity across the solution curve is discussed. Further, the evolution of a characteristic shock and the corresponding interaction with the weak discontinuity are studied. The amplitudes of the reflected wave, the transmitted wave and the jump in the shock acceleration influenced by the incident wave after interaction are evaluated. Finally, the influence of van der Waals excluded volume in the behavior of the weak discontinuity is completely characterized.

1. Introduction

Many physical phenomena in the field of astrophysics and hypersonic aerodynamics are modeled by first order quasilinear hyperbolic system of PDEs. A notable remark in this context is that the solutions to such a system of PDEs encounter elementary waves such as shock waves and weak discontinuity waves. For the safety assessments and prediction of disaster due to explosions point of view, one should have a clear understanding of the behavior of the solution of such a system of PDEs. In order to study such physical phenomenon completely, one has to solve the corresponding governing system of PDEs. But we do not have the luxury to solve these system of PDEs exactly. Hence, we rely on Lie group analysis which is one of the systematic and most powerful technique to solve such nonlinear system of PDEs (see [1–5]). It may be noted that understanding the behavior of the shock waves associated with such solutions has been a field of continuing research interest over the years. Unfortunately, most of the work done in the past on shock wave phenomena has been limited to ideal gas flows, while the effects of real-gas are significant for many flow systems. Real-gas effects can have a noticeable impact on the flow features, such as shock formation and shock stand off distance in a blunt body flow. Because of their importance, real-gas effects have recently been the focus of several studies. A detailed study to understand the wave interaction problem within the context of hyperbolic systems has been carried out by Jeffrey [6]. Further the extension of this work to elasticity and magneto-fluid dynamics is found in the work of Morro [7,8].

The evolutionary behavior of characteristic shock and its interaction with a weak discontinuity, together with the properties of incident, reflected and transmitted waves via Lie group analysis has been studied in [9,10]. In [11], the authors examined the effect of an incident wave to create a discontinuity in the acceleration of the shock. Further, they determined the amplitudes of reflected and transmitted waves as well as the jump in shock acceleration along the shock line. The work

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in [12] accounts for the time of shock formation for the fastest transmitted wave when it has overtaken and interacted with a shock wave in case of a polytropic gas. The interaction of a weak discontinuity with a shock wave in an axi-symmetric dusty gas flow can be found in [13,14]. Wave interactions in magnetogasdynamics and the corresponding Riemann problem have been discussed by Raja Sekhar and Sharma [15,16]. Radha et al. [17] discussed the interaction of shock waves with discontinuities. In [18] and [19], authors obtained some exact solutions to 3×3 quasilinear hyperbolic system of PDEs (simplified magnetogasdynamics) in Euler and Lagrangian coordinates, respectively. Either of the works did not consider the effect of magnetic field. In this article, we consider 4×4 quasilinear hyperbolic system of magnetogasdynamics describing a planar or cylindrically symmetric or spherically symmetric flows in a van der Waals gas, and use the symmetry group of transformations to determine a particular exact solution that exhibits space time dependence. With the help of this solution, we study the nonlinear wave interaction phenomenon influenced by the real gas effects, such as van der Waals excluded volume, and describe a complete history of evolutionary behavior of characteristic shock together with the incident, reflected and transmitted waves.

The structure of the paper is organized as follows: Section 2 describes Lie symmetry analysis for the governing system of PDEs with van der Waals gas equation of state. We reduce it to an autonomous system of PDEs and obtain an exact solution. In Section 3, the evolution of characteristic shock is studied using the Rankine–Hugoniot jump conditions. The evolution of C^1 discontinuity across the solution curve is discussed in Section 4. Section 5 deals with the interaction of the weak discontinuity with characteristic shock and derivation of amplitudes of the reflected and transmitted waves and the jump in shock acceleration influenced by the incident wave amplitude after interaction and conclusions are presented in Section 6.

2. Basic equations and their symmetry analysis

Before we introduce the set of equations considered in this paper, it is worth briefing the magnetohydrodynamic equations in a vector form (see [20]). Following usual notations which are explained below, the governing equations (PDEs) for the continuous motion of a non-ideal fluid in the absence of viscosity and thermal conduction can be written as

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{u} = -\frac{1}{\rho}\nabla(p + \frac{\mathbf{H}^2}{8\pi}) + \frac{1}{4\pi\rho}(\mathbf{H} \cdot \nabla)\mathbf{H}, \tag{2}$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p + \rho a^2 \nabla \cdot \mathbf{u} = 0, \tag{3}$$

$$\frac{\partial \mathbf{H}}{\partial t} + (\mathbf{u} \cdot \nabla)\,\mathbf{H} = (\mathbf{H} \cdot \nabla)\mathbf{H} - \mathbf{H}(\nabla \cdot \mathbf{u}), \tag{4}$$

$$\nabla \cdot \mathbf{H} = 0. \tag{5}$$

Here Eq. (1) is the conservation of mass in case of compressible flow and (2) represents conservation of momentum corresponding to the magnetohydrodynamic flow. The energy equation in terms of internal energy is given by (3). Eq. (4) represents the momentum balance corresponding to magnetic forces. Finally the magnetic field is divergence free as indicated in Eq. (5), where ρ is the fluid density, p the pressure, a the speed of sound, $\mathbf{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ the velocity vector and $\mathbf{H} = (H_1, H_2, H_3) \in \mathbb{R}^3$ the magnetic field vector. For a non-ideal fluid, $a = \sqrt{\frac{\gamma p}{\rho(1-b\rho)}}$, where γ is the specific heat ratio with $1 < \gamma < 2$ and b is the van der Waals excluded volume which is very small.

The above governing equations are more generic in nature, and depending on the geometry under consideration these get simplified. For the current investigation, we consider a unidirectional velocity along a fixed direction and the direction of the corresponding magnetic field is chosen in perpendicular direction to have an induced flow. Accordingly, we choose

$$\mathbf{u} = u(\xi, t)\hat{e}_i\delta_{1i}, \quad \mathbf{H} = H(\xi, t)\hat{e}_j\delta_{2j} \tag{6}$$

where δ_{ij} denotes the Kronecker delta, t denotes the time, ξ denotes the spatial co-ordinate and $\hat{e}_i$ denotes the unit vector in the ith direction. It may be noted that ξ and $\mathbf{e_i}$ depend on the co-ordinate system. For example in case of (x, y, z) Cartesian co-ordinates, we have $\xi = x$, $\hat{e}_1 = \hat{i}$, $\hat{e}_2 = \hat{j}$, $\hat{e}_3 = \hat{k}$, denoting unidirectional flow, and in case of cylindrical co-ordinates $\xi = r$, $\hat{e}_1 = \hat{e}_r$, $\hat{e}_2 = \hat{e}_\theta$, $\hat{e}_3 = \hat{e}_z$, denoting radial flow. Accordingly, the above set of governing equations under (6) take a simple form. The above considerations for the case of unidirectional flow allow us to write the governing equations in a