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Stochastic quasilinear viscoelastic wave equation with degenerate damping and source terms

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ABSTRACT

In this paper, we consider a stochastic quasilinear viscoelastic wave equation with degenerate damping and source term. We prove the blow-up of solution for stochastic quasilinear viscoelastic wave equation with positive probability or explosive in energy sense.

1. Introduction

In this paper, we are concerned with the following stochastic quasilinear viscoelastic wave equation with degenerate damping and source term

$$
\begin{aligned}
&|u_t(t)|^{\rho}u_{tt}(t) - \Delta u(t) - \Delta u_{tt}(t) + \int_0^t g(t-\tau)\Delta u(\tau)d\tau \\
&\quad +|u(t)|^k\partial j(u_t(t)) = |u(t)|^{p-1}u(t) + \epsilon\sigma(x,t)\partial_t W(x,t), \ \ (x,t)\in D\times[0,T), \qquad (1.1)\\
&u(x,t)=0, \ \ (x,t)\in \partial D\times[0,T),\\
&u(x,0)=u_0(x), \ \ u_t(x,0)=u_1(x)\, x\in \bar{D},
\end{aligned}
$$

where D is a bounded domain in $\mathbb{R}^n$ $(n\geq 1)$ with a smooth boundary ∂D, $k,p\geq 0$ and $g(\cdot):\mathbb{R}_+\to\mathbb{R}_+, j(\cdot):\mathbb{R}\to\mathbb{R}$ are given functions to be specified later. We use ∂j to denote its sub-differential (see [1]). Here, $W(x,t)$ is an infinite dimensional Wiener process and $\sigma(x,t)$ is $L^2(D)$ valued progressively measurable. Problems of this type arise in material science and physics and have been extensively studied, for example, see [1,2]. The cases on nonlinear viscoelastic wave equation problems have been considered by many authors, and several results concerning existence, nonexistence and asymptotic behavior have been established in [1–4] and the references there in. Under the consideration of random environment, there are many works on the stochastic wave equation with global existence and invariant measure for linear and degenerate damping (see references in [5–8]). For the nonlinear stochastic viscoelastic wave equation with linear damping, the authors have proved the global solutions and blow-up with positive probability for the stochastic viscoelastic wave equation (see

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in [5,9,8]). Han and Wang [2] investigated the following degenerate viscoelastic wave equation

$$
\begin{aligned}
&u_{tt}(t) - \Delta u(t) + \int_0^t g(t-s)\Delta u(s)ds + |u(t)|^k \partial j(u_t(t)) \\
&\qquad = |u(t)|^{p-1}u(t) \ \text{ in } \Omega \times (0,T), \\
&u(x,t) = 0 \ \text{ on } \partial\Omega \times (0,T), \\
&u(x,0) = u_0(x), \ u_t(x,0) = u_1(x) \ \text{ in } \Omega,
\end{aligned}
$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^n$, $k, p \geq 0$, and $g(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+, j(\cdot) : \mathbb{R} \to \mathbb{R}$ are given functions to be specified later. They prove the global existence and blow-up of solution for this problem. Cheng et al. [5] consider the stochastic viscoelastic wave equation with nonlinear damping and source term

$$
\begin{aligned}
&u_{tt}(t) - \Delta u(t) + \int_0^t h(t-\tau)\Delta u(\tau)d\tau + |u_t(t)|^{q-2}u_t(t) \\
&\quad = |u(t)|^{p-2}u(t) + \epsilon\sigma(x,t)\partial_t W(x,t), \quad \text{in } D \times [0,T], \\
&u(x,t) = 0 \text{ on } \partial D \times [0,T], \\
&u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \text{ in } D,
\end{aligned}
$$

where D is a bounded domain with smooth boundary $\partial D, q, p \geq 2, \epsilon$ is a given positive constant which measures the strength of noise, and $W(x,t)$ is an infinite dimensional Wiener process, $\sigma(x,t)$ is $L^2(D)$-valued and h is a positive relaxation function. In this paper, the authors studied the local solution of stochastic viscoelastic wave equation and investigated the solution blow-up with positive probability or it is explosive in energy sense in $p > q$. Moreover, Rana et al. [9] proved the global existence and finite time blow-up in a class of stochastic nonlinear wave equations form

$$
\begin{aligned}
&\partial_{tt}u(t) - \Delta\partial_t u(t) - div(|\nabla u(t)|^{\alpha-1}\nabla u(t)) - div(|\nabla\partial_t u(t)|^{\beta-2}\nabla\partial_t u(t)) \\
&\qquad + a|\partial_t u(t)|^{q-2}\partial_t u(t) = b|u(t)|^{p-2}u(t) + \sigma(x,t)\partial_t W(x,t), \quad \text{in } D \times [0,T), \\
&u(x,t) = 0 \text{ on } \partial D \times (0,T), \\
&u(x,0) = u_0(x), \quad \partial_t u(x,0) = u_1(x) \text{ in } D,
\end{aligned}
$$

where $D \subset \mathbb{R}^n$ is a bounded domain with smooth boundary ∂D. Motivated by previous works, we study the blow-up of solution for stochastic quasilinear viscoelastic wave equation (1.1) with positive probability or explosive in energy sense.

2. Preliminaries

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $\mathbf{B}(X)$, and let $(\Omega, \mathfrak{F}, P)$ be a probability space. We set $H = L^2(D)$ with the inner product and norm denoted by $(\cdot,\cdot)$ and $\|\cdot\|$, respectively. Denote by $\|\cdot\|_q$ the $L^q(D)$ norm for $1 \leq q \leq \infty$ and by $\|\nabla\cdot\|$ the Dirichlet norm in $V = H_0^1(D)$ which is equivalent to $H^1(D)$ norm.

Now, we make the following assumptions:

(H1) $u_0(x) \in H_0^1(\Omega), \quad u_1(x) \in L^2(D)$.

(H2) $0 \leq \rho \leq \frac{2}{n-2}$ if $n \geq 3, \quad 0 < \rho < \infty$ if $n = 1, 2$.

(H3) $j(\cdot) : \mathbb{R} \to \mathbb{R}$ be a continuous, convex real valued function, $k, m, p \geq 0$,

$$k \leq \frac{n}{n-2}, \quad p+1 \leq \frac{2n}{n-2} \quad \text{if} \quad n \geq 3. \tag{2.1}$$

In addition, there exist positive constants c_0, c_1, c_2, c_3 such that for all $s, v \in \mathbb{R}, j(\cdot)$ satisfies

(i) coercivity: $j(s) \geq c_0|s|^{m+1}$,

(ii) strict monotonicity : $(\partial j(s) - \partial j(v))(s-v) \geq c_1|s-v|^{m+1}$,

(iii) continuity: $\partial j(s)$ is single valued and $|\partial j(s)| \leq c_2|s|^m + c_3$.

(H4)

$$p \leq \max\{\frac{p^*}{2}, \frac{p^*m+k}{m+1}\}, \quad p^* = \frac{2n}{n-2}. \tag{2.2}$$

(H5) $g : \mathbb{R}^+ \to \mathbb{R}^+$ is a bounded nonincreasing C^1 function satisfying

$$g(s) > 0, \quad 1 - \int_0^\infty g(s)ds = l > 0$$

and there exist positive constants ξ_1 and ξ_2 such that

$$-\xi_1 g(t) \leq g'(t) \leq \xi_2 g(t), \quad t \geq 0. \tag{2.3}$$

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