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Periodic conjugate direction algorithm for symmetric periodic solutions of general coupled periodic matrix equations

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ABSTRACT

Analysis and design of linear periodic control systems are closely related to the periodic matrix equations. The conjugate direction (CD) method is a famous iterative algorithm to find the solution to nonsymmetric linear systems $Ax = b$. In this work, a new method based on the CD method is proposed for computing the symmetric periodic solutions $(X_1, X_2, \dots, X_\lambda)$ and $(Y_1, Y_2, \dots, Y_\lambda)$ of general coupled periodic matrix equations

$$\begin{cases} \sum_{s=0}^{\lambda-1}\left(A_{i,s}X_{i+s}B_{i,s} + C_{i,s}Y_{i+s}D_{i,s}\right) = M_i, \\ \sum_{s=0}^{\lambda-1}\left(E_{i,s}X_{i+s}F_{i,s} + G_{i,s}Y_{i+s}H_{i,s}\right) = N_i, \end{cases}$$

for $i = 1, 2, \dots, \lambda$. The key idea of the scheme is to extend the CD method by means of Kronecker product and vectorization operator. In order to assess the convergence properties of the method, some theoretical results are given. Finally two numerical examples are included to illustrate the efficiency and effectiveness of the method.

1. Prologue

The major objective of the work presented in this paper is to generalize the CD method to compute the symmetric periodic solutions $(X_1, X_2, \dots, X_\lambda)$ and $(Y_1, Y_2, \dots, Y_\lambda)$ of general coupled periodic matrix equations

$$\begin{cases} \sum_{s=0}^{\lambda-1}\left(A_{i,s}X_{i+s}B_{i,s} + C_{i,s}Y_{i+s}D_{i,s}\right) = M_i, \\ \sum_{s=0}^{\lambda-1}\left(E_{i,s}X_{i+s}F_{i,s} + G_{i,s}Y_{i+s}H_{i,s}\right) = N_i, \end{cases} \tag{1.1}$$

for $i = 1, 2, \dots, \lambda$ where the coefficient matrices $A_{i,s}, C_{i,s} \in \mathrm{R}^{p\times n}$, $E_{i,s}, G_{i,s} \in \mathrm{R}^{s\times n}$, $B_{i,s}, D_{i,s} \in \mathrm{R}^{n\times q}$, $F_{i,s}, H_{i,s} \in \mathrm{R}^{n\times t}$, $M_i \in \mathrm{R}^{p\times q}$, $N_i \in \mathrm{R}^{s\times t}$, and the symmetric solutions $X_i, Y_i \in \mathrm{SR}^{n\times n}$ are periodic with period λ, i.e., $A_{i+\lambda,s} = A_{i,s}$, $B_{i+\lambda,s} = B_{i,s}$, $C_{i+\lambda,s} = C_{i,s}$, $D_{i+\lambda,s} = D_{i,s}$, $M_{i+\lambda} = M_i$, $E_{i+\lambda,s} = E_{i,s}$, $F_{i+\lambda,s} = F_{i,s}$, $G_{i+\lambda,s} = G_{i,s}$, $H_{i+\lambda,s} = H_{i,s}$, $N_{i+\lambda} = N_i$, $X_{i+\lambda} = X_i$ and $Y_{i+\lambda} = Y_i$ for $s = 1, 2, \dots, \lambda - 1$ and $i = 1, 2, \dots$. It is worth noting that all previous papers related to periodic matrix equations considered the periodic matrix equations with $s = 0, 1$. But general coupled periodic matrix equations (1.1) present a new kind of periodic matrix equations that can be used in future works of control and system theory.

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The periodic matrix equations have been used in the literature to analyze and design of linear periodic descriptor systems arising in applications [1,2]. For instance, the cyclic lifted representation of linear discrete-time periodic descriptor system with time-varying dimensions

$$E_i x_{i+1} = A_i x_i + B_i u_i, \quad y_i = C_i x_i, \quad i = 1, 2, \ldots, \tag{1.2}$$

which plays an important role in extending many theoretical results and numerical algorithms for this system to the periodic setting [3–5] can be given by the following coupled periodic matrix equations [6]

$$EX_{i+1} = AX_i + BU_i, \quad Y_i = CX_i, \quad i = 1, 2, \ldots. \tag{1.3}$$

When Newton's method is used to solve the discrete periodic Riccati matrix equation, each step of this method involves finding the solution of a reverse-time discrete periodic Lyapunov matrix equation [7]. For characterizing the complete reachability and observability for the periodic discrete-time descriptor systems (1.2), we need to solve the projected generalized discrete-time periodic Lyapunov equations in the form [8]

$$\begin{cases} A_i X_i A_i^T - E_i X_{i+1} E_i^T = Q_l(i) B_i B_i^T Q_l(i)^T, \\ X_i = Q_r(i) G_i Q_r(i)^T, \end{cases} \qquad i = 1, 2, \ldots. \tag{1.4}$$

The semi-global stabilization problem of discrete-time linear periodic system needs to find a solution of discrete-time periodic Lyapunov matrix equation [9]. When dealing with design of periodic Luenberger observers for the linear discrete-time periodic systems, the periodic Sylvester matrix equation is encountered [10].

In recent years due to applications of (periodic) matrix equations, the area of numerical methods for solving matrix equations have expanded at a fast pace [10–22]. The numerical methods to solve periodic matrix equations can be classified in two main categories: direct methods and iterative methods [23]. Direct methods are not applicable to compute the solutions of matrix equations because of their high computational cost and huge memory requirements. Lv and Zhang proposed explicit and complete parametric solutions to the discrete periodic Sylvester matrix equations

$$F_i X_i - X_{i+1} A_i = -G_i C_i, \quad i = 1, 2, \ldots, \tag{1.5}$$

without any constraints on the coefficient matrices [10]. Andersson et al. introduced parallel algorithms based on the periodic Schur decomposition for triangular periodic Sylvester matrix equations [24]. A large part of research in solving linear matrix equations has been aimed towards the generalization of iterative methods proposed to solve nonsymmetric systems $Ax = b$ [25–31]. In [32,33], low-rank versions of alternating direction implicit method and the Smith method were proposed to the solutions of projected periodic Lyapunov equations in lifted form with low-rank right-hand side. In [34], a gradient based iterative (GI) algorithm was proposed for finding the generalized reflexive solutions of the general coupled discrete time periodic matrix equations. Hajarian proposed an iterative method based on the idea of conjugate gradient method on the normal equations (CGNE) for solving the periodic continuous-time generalized coupled Sylvester matrix equations [35]. In [36], two iterative algorithms were established to solve the discrete-time periodic Lyapunov matrix equations by extending CGNE method. Recently in [37–40], matrix forms of BCR, Bi-CGSTAB, CGS, BiCOR and CORS methods were introduced for finding solutions of several periodic matrix equations.

The layout of the remainder of this article is as follows. After a brief introduction to the CD method and solvability of (1.1), the extension of the CD method for calculating the symmetric periodic solutions of (1.1) is described in Section 2. In Section 3, convergence properties of new method are given. Section 4 presents two numerical examples and Section 5 provides conclusions.

2. CD method and its extension to (1.1)

We start from the problem of general coupled periodic matrix equations (1.1) over symmetric periodic matrices. This problem can be equivalently transformed into the following coupled Sylvester matrix equations

$$\begin{cases} \sum_{s=0}^{\lambda-1}\left(A_s X B_s + C_s Y D_s\right) = M, \\ \sum_{s=0}^{\lambda-1}\left(B_s^T X A_s^T + D_s^T Y C_s^T\right) = M^T, \\ \sum_{s=0}^{\lambda-1}\left(E_s X F_s + G_s Y H_s\right) = N, \\ \sum_{s=0}^{\lambda-1}\left(F_s^T X E_s^T + H_s^T Y G_s^T\right) = N^T, \end{cases} \tag{2.1}$$

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