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Depth-averaged Lattice Boltzmann and Finite Element methods for single-phase flows in fractures with obstacles

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ABSTRACT

We use Lattice Boltzmann Method (LBM) MRT and Cumulant schemes to study the performance and accuracy of single-phase flow modeling for propped fractures. The simulations are run using both the two- and three-dimensional Stokes equations, and a 2.5D Stokes–Brinkman approximate model. The LBM results are validated against Finite Element Method (FEM) simulations and an analytical solution to the Stokes–Brinkman flow around an isolated circular obstacle. Both LBM and FEM 2.5D Stokes–Brinkman models are able to reproduce the analytical solution for an isolated circular obstacle. In the case of 2D Stokes and 2.5D Stokes–Brinkman models, the differences between the extrapolated fracture permeabilities obtained with LBM and FEM simulations for fractures with multiple obstacles are below 1%. The differences between the fracture permeabilities computed using 3D Stokes LBM and FEM simulations are below 8% . The differences between the 3D Stokes and 2.5 Stokes–Brinkman results are less than 7% for FEM study, and 8% for the LBM case. The velocity perturbations that are introduced around the obstacles are not fully captured by the parabolic velocity profile inherent to the 2.5D Stokes–Brinkman model.

1. Introduction

Hydraulic fracturing is widely used in oil & gas industry to improve hydrocarbons extraction from low-permeability matrix rocks such as shales [1]. It is also practiced to enhance heat extraction from sedimentary [2] and crystalline rocks [3]. Propping agents such as sand grains are injected into hydraulic fractures to counteract the mechanical operation of high wall pressures, which tend to reduce fracture aperture. The achieved distribution of proppant grains may have a large impact on the technological flows, which control transport processes in the hydraulic fractures and determine the exchange surface between the fracture network and the surrounding, low-permeability rock matrix. Fracture wall asperities play the role of proppant grains and maintain the aperture in natural rock fractures, which are dominant fluid pathways at great depths. The complex flow phenomena in propped fractures and their effective transport properties have been extensively investigated using lab experiments [4,5], field observations [6], and numerical simulations [4]. Flow through apertures with cylindrical obstacles is also relevant to engineering applications such as micropillar arrays of cylinders used in Lab-on-a-chip systems for high performance liquid chromatography [7].

The Lattice Boltzmann Method (LBM) is a weakly-compressible numerical method conceptually based on statistical thermodynamics and Boltzmann equation. The method is most commonly used to solve the Navier–Stokes equation, but it was successfully applied to problems that range from the Poisson or Poison–Boltzmann equations [8,9], through the

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Nernst–Planck [10] to plain convection equations [11]. The performance of LBM for incompressible flows range from proper path to approach low Mach number limit [12], development of general [13] and axisymmetric [14] schemes specifically for incompressible flows, and investigation of numerical techniques meant to avoid truncation error accumulation [15]. This work is concentrated on structured grid based LBM, but flow properties of random domains were also investigated using an unstructured grid variant of the method [16].

LBM is a relatively new method [17–19] and attracts an increasing amount of attention, which has led to its rapid development in recent years. In Earth Sciences Lattice Boltzmann Method has been used in permeability calculation, for both single and multiple phase problems. The method itself historically origins from cellular automaton fluid models (lattice automata, LCGA models [20]). Robustness of the lattice-based models for permeability computations was foreseen in the method early days [21]. Evolution of the modern formulation of the LBM [22,23] led to permeability studies of random medium [24], including natural one [25]. Recent studies includes digital rock physics benchmarking study [26], where 3D tomography images were used to estimate permeability using LBM, among other rock properties calculated by using other methods. Porous media flows in general (especially multiphase or multiphysics cases) are one of the application areas that is often listed as well suited for LBM [27]. Prediction of pressure drop in such domains for various regimes were studied in a number of papers, including detailed experimental verifications using water tunnel and μ-CT imaging [28]. Numerical investigation employing Lattice Boltzmann Method include, but are not limited to, artificially generated fibrous geometry [29], nanoscale simulations [30,31] in low Knudsen number regime, investigations of scanned and generated geometries [32,28,33].

The Finite Element Method (FEM) is a well established numerical method for discretizing the weak integral formulation of partial differential equations. The method was originally developed for complex engineering problems of engineering applications in structural analysis, but it has been extended to a wide range of problems of mathematical physics. Using FEM for the incompressible Stokes problem requires some tailored approaches such as the mixed formulation [34] and dedicated solution schemes [35]. In Earth Sciences, FEM has been extensively used to study various deformation problems in heterogeneous media such as rock folding [36–38], earth mantle convection [39,40], shear zone dynamics [41,42], and particle suspension flows [43,44]. FEM simulations have been also developed for purely hydrodynamic problems in geology. Most simulations are performed in the framework of upscaled, Darcy-type models for problems such as groundwater flow [45,46], fracture and fracture network flow [47], multiphase flow in fractured porous media [48,49], thermal and thermohaline convection in porous media [50–53], and hydrocarbon migration [54–56]. With the advent of computer power and numerical method development, direct numerical simulations of single- and multiphase flow problems in fractures [57], porous matrix [58–60], and dual-porosity media such as karst reservoirs [61,62], arise nowadays.

Direct numerical simulations of three dimensional fluid flows are computationally expensive and reduced-order, 2.5D numerical models have been proposed for flow through thin, plane-walled domains. Holme and Rothman [63] presented a LBM study of miscible two-phase flows in a Hele-Shaw cell, including the effects of viscous drag due to the no-slip boundary conditions at the plates. Flekkøy et al. [64] used a LBM model for the flow and tracer transport in the central layer of a Hele-Shaw cell. The viscous drag effect was included by using a two-dimensional Brinkman-type equation for the mid-plane velocity, which was derived by assuming a parabolic velocity profile across the cell. Using an analytical solution for a rectangular channel as a benchmark, the error due to the approximation was estimated at less than 10% and the authors claimed that the simulations are expected to overestimate viscous drag close to the sidewalls. Clague et al. [65] used a three-dimensional LBM to study hydraulic permeability of bounded and unbounded fibrous media, with either ordered and disordered structure. The numerical results for the bounded disordered media were shown to be in a good agreement with the theoretical predictions of Tsay and Weinbaum [66], who developed an effective medium approach to the problem based on a Brinkman-type equation derived by an in-plane rather than depth averaging.

Depth-averaging is often used to thread viscous coupling due to the plates in analytical studies of the flow in Hele-Shaw cells. For example, Fernandez et al. [67] presented a linear stability analysis of viscous fingering inside the Hele-Shaw cell based on an approximate two-dimensional Brinkman equation for the depth-averaged velocity. An LBM approach to viscous fingering in Hele-Shaw cells based on depth-averaging was used by Grosfils et al. [68]. Venturoli and Boek [69] used the same method in LBM simulations to analyze single phase flow in pseudo-2D capillary network models. Single and multiphase flow properties of realistic rock geometries were studied in another work of Boek and Venturoli [32]. Works by Hale et al. [7] and other by Hong et al. [70] are dedicated to multiphase flow in Hele-Shaw cell, mainly capillary action for thin domain with boundaries. The depth-averaged Stokes–Brinkman equation was used by Horgue et al. [71] and compared with experimental results for two-phase flows in the domain with cylindrical obstacles. Nagel et al. [72] used Boundary Element Method to solve the 2.5D Stokes–Brinkman model for a deformable droplet in a microfluidic flow in shallow channel. Laleian et al. [73] studied the accuracy of the depth-averaged 2.5D model for microfluidic devices with variable aperture. The computational speedup up to 40 times with respect to 3D simulations was reported along with an accuracy of 10% in terms of permeability.

In the present work, we introduce a 2.5D Stokes–Brinkman fracture flow model into MILAMIN [74], a FEM-based incompressible Stokes solver programmed in MATLAB, and TCLB [75,76], a Lattice Boltzmann method based CUDA GPU code for Navier–Stokes problems. The 2.5D codes were validated against an analytical solution for the Stokes–Brinkman flow around a single circular obstacle and cross-checked against each other for systems with multiple obstacles. Fully resolved solutions were also obtained using the 3D variants of the two codes and we present a systematic comparison between the fully resolved 3D and the reduced 2.5D solutions to evaluate the accuracy and robustness of the approximate 2.5D Stokes–Brinkman fracture flow model.