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Dual analysis for heat exchange: Application to thermal bridges

Samer Taoum *, Emmanuel Lefrançois

Laboratoire Roberval, CNRS UMR 7337, Sorbonne Universités - Université de Technologie de Compiègne, CS 60319, 60203 Compiègne, France

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ABSTRACT

The research presented in this article is a contribution to the characterization of thermal dual analysis.

The aim of a dual analysis based on two complementary numerical methods is to calculate the stationary temperature field. The two methods are respectively the classical *Kinetically Admissible (KA)* finite element or temperature method, and the *Statically Admissible (SA)* flow method.

The originality of this work is twofold. First, it uses a flow approach (SA) in 2D and 3D that naturally respects flow conservation (in contrast to the KA method). Second, calculating the dissipation energy allows the exact solution of a problem to be enclosed, with the lower and upper bounds being calculated respectively by the KA and SA methods.

Thermal bridges are areas of risk in the building walls, because they can give rise to uncontrolled increments in heat transfer. A number of studies have looked at thermal bridges, but much of the numerical code for building energy simulations uses heat transfer models based on one-dimensional heat flow analysis. Fabrizio Ascione (2014) showed that this can lead to unreliable results. The originality of this paper is to consider 2D and 3D thermal bridges using dual analysis with mesh sizes smaller than those used in conventional finite element approaches.

1. Introduction

In finite element analysis, solving a physical problem using the dual approach consists in discretizing two conjugated fields, temperature (*Kinetically Admissible*, or *KA* method) and heat flow (*Statically Admissible*, or *SA* method), in order to obtain approximations that enclose the unknown exact solution. The KA method corresponds to the classical finite element method [1]. The SA method is presented and described in this paper. Dual analysis goes back a long way, but more recently it was developed by B.M Fraeijs de Veubeke & M.A Hogge at the University of Liège, Belgium [2]. It is an approach that has often been misunderstood and that has tended to be restricted to a small community of specialists. It has been widely used in solid and elastic mechanics [3–8], but rarely for thermal applications. The main objective of this paper is to make the concept clearer for users of the finite element method.

To solve the thermal problem, it is sufficient to discretize two fields: temperature and heat flow. By minimizing the energy of the system with respect to the parameters of the chosen approximation, we obtain an integral form that must respond to the conjugated field [2,9–11,7]. In this way, a functional energy is provided: if dependent on temperature it corresponds to

* Corresponding author.
E-mail address: samer.taoum@utc.fr (S. Taoum).

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the weak form of the KA method, and if dependent on heat flow it corresponds to the weak form of the SA method for the equilibrium equations for heat flow.

The main advantage of dual analysis is using the same mesh to perform two separate analyses that enclose the exact solution. The SA method's strength is that it respects the continuity of flow, in contrast to the KA method, which respects this principle only as Δx tends to 0 [10,11,7].

Since the KA method is already well defined and used [1], our aim here is to present and describe the SA method in more or less general terms for the 2D and 3D cases and for any approximation order of discretization. So as to make it accessible, this work uses MATLAB code. The proposed formulation can be used to highlight the properties of dual analysis. It is a formulation suitable for academic problems, but that also may be enhanced for tackling more complex problems. We address two academic problems (2D and 3D), then present a thermal bridge application in 2D and 3D.

Heat in a building will "short circuit" through an element that has a much higher conductivity than surrounding material. This phenomenon is called "thermal bridging". Numerical code for energy simulation in buildings often uses heat transfer models based on a one-dimensional heat flow: this can give unreliable results [12]. TRNSYS building energy simulation programs, for example, are based on solving the equations governing heat transfer by considering only a 1D flow, while programs such as ESP, using finite difference methods, solve heat transfer equations in 3D [13]. The simulation of thermal bridges varies according to the particular tool: it will either be through variation of the surface transmission coefficient of the wall, or through variation of the surface or the specific transmission coefficient [14]. The effectiveness of powerful programs such as DOE and EnergyPlus, used for energy audits of buildings, may be reduced if thermal bridges are not properly evaluated [15]. Many programs still apply one-dimensional heat transfer algorithms, even though a variety of research and optimization problems require very accurate methods of computation.

Within the general context of an energy saving environment and the sustainable development of buildings, this article presents applications that use dual analysis to address a thermal bridge developed in [16,17].

2. Mathematical model

Consider a domain V enclosed by a boundary S. The thermal equilibrium is governed by the heat equation completed by Fourier's law:

$$\vec{\nabla} \cdot \vec{q} = f_v \quad \text{in} \quad V \quad \text{with} \quad \vec{q} = -\kappa \vec{\nabla} T, \tag{1}$$

where $\vec{q}$ [W/m^2] is the flow, $T(x, y, z)$ the temperature, κ [W/m/K] the thermal conductivity of the material and f_v [W/m^3] is a source term.

In order to achieve the kind of mathematically well-posed problem illustrated Fig. 1, these relations are supplemented with boundary conditions applied to S. They are classified into three types:

1. Dirichlet condition (imposed temperature): $T = T_0$.
2. Neumann condition (imposed flow): $\vec{q}.\vec{n} = \varphi$.
3. Cauchy condition (convective exchange): $\overrightarrow{q} . \overrightarrow{n} = h(T(s) - T_{air})$.

The vector $\vec{n}$ denotes the external normal to the wall, h [W/m^2/K] and T_{air} are respectively the heat cœfficient transfer (HCT) and the ambient temperature.

The KA method is not described in this article. For details see [1]. The functional energy of the KA method is:

$$\mathscr{E}(T) = \underbrace{\frac{1}{2} \iiint_V \vec{\nabla} T^T [\kappa] \vec{\nabla} T dV - \iiint_V f_v T dV}_{\mathscr{E}_{int}} - \underbrace{\oint_S \kappa \vec{\nabla} T . \vec{n} T ds}_{\mathscr{E}_{ext}}. \tag{2}$$

The SA method is based on the discretization of a functional energy in terms of flow [2,11,18,7]:

$$\mathscr{E}(q) = \underbrace{\frac{1}{2} \iiint_V \vec{q}^T [\kappa]^{-1} \vec{q} dV - \iiint_V f_v T dV}_{\mathscr{E}_{int}} + \underbrace{\oint_S \vec{q}.\vec{n} T ds}_{\mathscr{E}_{ext}}, \tag{3}$$

where $[\kappa]$ is the matrix of the conductivity for orthotropic materials. The discretization of this functional is detailed in the next section.

3. The SA method's numerical model

The SA method requires the discretization of the domain in the form of a mesh composed of *nelt* elements, such that the functional energy is:

$$\mathscr{E}(q) = \sum_{e}^{nelt} \mathscr{E}^e(q). \tag{4}$$

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