Computers and Mathematics with Applications 75 (2018) 3571–3588

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Modal Hermite spectral collocation method for solving multi-dimensional hyperbolic telegraph equations

Bashar Zogheib [a], Emran Tohidi [b,*]

[a] Department of Mathematics and Natural Sciences, American University of Kuwait, Kuwait
[b] Department of Mathematics, Kosar University of Bojnord, P. O. Box 9415615458, Bojnord, Iran

ARTICLE INFO

Article history:
Received 7 August 2017
Received in revised form 10 November 2017
Accepted 11 February 2018
Available online 7 May 2018

Keywords:
Telegraph equation
Spectral collocation method
Hermite polynomials
Operational matrices
LU factorization

ABSTRACT

The present research is contemplated proposing a numerical solution of multi-dimensional hyperbolic telegraph equations with appropriate initial time and boundary space conditions. The truncated Hermite series with unknown coefficients are used for approximating the solution in both of the spatial and temporal variables. The basic idea for discretizing the considered one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) telegraph equations is based on the collocation method together with the Hermite operational matrices of derivatives. The resulted systems of linear algebraic equations are solved by some efficient methods such as LU factorization. The solution of the algebraic system contains the coefficients of the truncated Hermite series. Numerical experiments are provided to illustrate the accuracy and efficiency of the presented numerical scheme. Comparisons of numerical results associated to the proposed method with some of the existing numerical methods confirm that the method is accurate and fast experimentally.

1. Introduction

In real world, multi-dimensional partial differential equations (PDEs) are more favorable than the one-dimensional PDEs for modeling the existing phenomena in science and engineering. Drying procedure in food engineering may be a typical instance in this regard [1]. Neglecting any spatial dimension can affect on the accuracy of the model for describing the physical and chemical events. Therefore, two-dimensional (2D) and three-dimensional (3D) PDEs are considered by researchers to model and simulate the aforementioned events. Among the differential models, parabolic PDEs can describe a phenomena by using some physics laws, but sometimes it may be better modeled by hyperbolic PDEs [2]. For instance, the experimental investigations in [3] seem to be better modeled by telegraph equation rather than the heat equation. It should be noted that, in the context of PDEs, the expression "hyperbolic" usually denotes to a special category of second-order equations [4]. Other applications of hyperbolic PDEs can be found in ecological and cosmological phenomena [5]. Because of the importance of these types of equations, robust and efficient tools should be explored to compute the solutions for simulating the events accurately.

In recent years, analytical approaches such as the Adomian decomposition method (ADM) [6], homotopy perturbation method (HPM) [7], variational iteration method (VIM) [8], homotopy analysis method (HAM) [9] and differential transform method (DTM) [10] have been proposed for solving hyperbolic PDEs. One of the main disadvantages of these methods is that, the boundary conditions for the spatial variables are neglected. It should be noted that, these assumptions make the problems to be artificial. In real world applications, specially in second order hyperbolic PDEs such as telegraph equations,

* Corresponding author.
E-mail address: emrantohidi@gmail.com (E. Tohidi).

https://doi.org/10.1016/j.camwa.2018.02.018
0898-1221/

we should impose some boundary limitations on the space variables. However, some few research works are devoted to the implementation of analytical methods for solving hyperbolic PDEs with boundary space conditions [11]. On the other hand, direct symbolic differentiation and integration in iterations of these schemes make them time consuming. This is another disadvantage of analytical methods for solving hyperbolic PDEs. For solving this issue (or drawback), the researchers focused on the numerical methods. In numerical methods, direct symbolic differentiation and integration are replaced by the operational matrices of differentiation and quadrature rules, respectively.

As a primary numerical method for solving hyperbolic PDEs, one can point out to the finite difference methods (FDMs). Conditional stability of explicit FDMs and need to large amount of computational time in implicit FDMs limit the applicability of this class of local numerical techniques [12]. Finite element methods (FEMs) [13] and finite volume methods (FVMs) [14] are other local popular methods for solving PDEs. Specially FVMs are more suitable than FDMs and FEMs for solving hyperbolic PDEs. But, implementation of them for 2D and 3D equations is complicated even for solving linear ones [14]. Similar to the analytical approaches [6–9], neglecting the boundary space conditions and imposing boundary time conditions instead of initial time conditions can be found in some research works that have been investigated by numerical methods such as the Taylor matrix method [15], Legendre multi-wavelet Galerkin method [16] and Chebyshev wavelet scheme [17]. Among the numerical methods that were proposed for solving hyperbolic telegraph equations with real boundary space and initial time conditions, one can point out to the compact FDM [18], Tau method [19], dual reciprocity boundary integral equation method [20], wavelet method [21], interpolation scaling function [22] and differential quadrature method [23]. Also, redefined extended cubic B-spline basis functions for approximating the space variable together with a FDM for discretizing the temporal variable were used for solving the linear 1D telegraph equations in [24]. Moreover, authors of [25] considered the method of lines (MOL) for solving the 3D telegraph equations, in which the B-spline functions were used for localizing the space variables and the resulted system of ordinary differential equations (ODEs) were solved by a fourth stage Runge–Kutta scheme.

Among the weighted residual methods, collocation method is a powerful tool for approximating the solution of hyperbolic PDEs that needs less computational time with respect to the Tau and Galerkin methods. Because in this method; we do not need to evaluate any integral terms or approximate any known function in equations [26,27]. Recently several research articles have considered the matrix and collocation methods for solving 1D hyperbolic telegraph equations such as Taylor matrix method [28] and Bessel collocation approach [29]. In this paper, we will present the space–time Hermite spectral collocation method for solving 1D, 2D and 3D telegraph equations in modal form. It should be noted that, space–time collocation methods are very popular with respect to the MOL methods among the mathematicians in solving multi-dimensional PDEs recently [27,30]. In this paper, we will approximate the solution of the following linear 1D telegraph equation

$$\frac{\partial^2\omega}{\partial\tau^2}+\alpha\frac{\partial\omega}{\partial\tau}+\beta\omega=\frac{\partial^2\omega}{\partial\xi^2}+g(\xi,\tau),\qquad(\xi,\tau)\in[0,X]\times[0,T],\tag{1}$$

with the initial time conditions

$$\omega(\xi,0)=h_0(\xi),\qquad\omega_\tau(\xi,0)=h_1(\xi),\qquad\xi\in[0,X],\tag{2}$$

and the Dirichlet boundary space conditions

$$\omega(0,\tau)=k_0(\tau),\qquad\omega(X,\tau)=k_1(\tau),\qquad\tau\in[0,T].\tag{3}$$

Also, we will consider the following linear 2D telegraph equation

$$\frac{\partial^2\omega}{\partial\tau^2}+\alpha\frac{\partial\omega}{\partial\tau}+\beta\omega=\frac{\partial^2\omega}{\partial\xi^2}+\frac{\partial^2\omega}{\partial\eta^2}+g(\xi,\eta,\tau),\qquad(\xi,\eta,\tau)\in[0,X]\times[0,Y]\times[0,T],\tag{4}$$

with the initial temporal conditions

$$\omega(\xi,\eta,0)=h_0(\xi,\eta),\qquad\omega_\tau(\xi,\eta,0)=h_1(\xi,\eta),\qquad(\xi,\eta)\in[0,X]\times[0,Y],\tag{5}$$

together with the Dirichlet boundary spatial conditions

$$\begin{aligned}&\omega(0,\eta,\tau)=k_0(\eta,\tau),\qquad\omega(X,\eta,\tau)=k_1(\eta,\tau),\qquad(\eta,\tau)\in[0,Y]\times[0,T],\\&\omega(\xi,0,\tau)=f_0(\xi,\tau),\qquad\omega(\xi,Y,\tau)=f_1(\xi,\tau),\qquad(\xi,\tau)\in[0,X]\times[0,T].\end{aligned}\tag{6}$$

Moreover, we will approximate the solution of the following linear 3D telegraph equation

$$\frac{\partial^2\omega}{\partial\tau^2}+\alpha\frac{\partial\omega}{\partial\tau}+\beta\omega=\frac{\partial^2\omega}{\partial\xi^2}+\frac{\partial^2\omega}{\partial\eta^2}+\frac{\partial^2\omega}{\partial\gamma^2}+g(\xi,\eta,\gamma,\tau),\qquad(\xi,\eta,\gamma,\tau)\in[0,X]\times[0,Y]\times[0,Z]\times[0,T],\tag{7}$$

with the initial time conditions

$$\omega(\xi,\eta,\gamma,0)=h_0(\xi,\eta,\gamma),\qquad\omega_\tau(\xi,\eta,\gamma,0)=h_1(\xi,\eta,\gamma),\qquad(\xi,\eta,\gamma)\in[0,X]\times[0,Y]\times[0,Z],\tag{8}$$