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Numerical analysis of a finite volume scheme for two incompressible phase flow with dynamic capillary pressure

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ABSTRACT

In this paper, we propose and analyze the convergence of a TPFA (Two Points Flux Approximation) finite volume scheme to approximate the two incompressible phase flow with dynamic capillary pressure in porous media. The fully implicit scheme is based on nonstandard approximation on mobilities and capillary pressure on the dual mesh. We derive a discrete variational formulation and we present a new result of convergence in a two or three dimensional porous medium. In comparison with static capillary pressure, the non-equilibrium capillary model requires more powerful techniques; especially the discrete energy estimates are not standard.

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1. Introduction

The capillary pressure and capillary action play a central role in the description of multiphase flow in porous media [1]. The dependence of capillary pressure–saturation curves on the history of the flow is known as capillary pressure hysteresis; the dependence of the capillary curves on the rate of change of saturation is due to capillary effects [2]. The dynamic (or non-equilibrium) capillary pressure relationship proposed by Stauffer [3], Hassanizadeh and Gray [1] has received much attention. They suggest that the dynamic capillary pressure is larger than static capillary pressure in drainage and smaller in wetting. This supposition has been found true by a number of experimental works. So, for non-equilibrium conditions, Hassanizadeh and Gray [4,1] proposed that the phases pressure difference can be written as a combination of the capillary pressure under equilibrium condition (static capillary pressure) and the product of the damping coefficient (τ) and the saturation rate.

Mathematical analysis, existence and uniqueness of weak solutions are given in Van Duijn et al. [5], Spayd and Shearer [6] for the Buckley–Leverett equation with capillary dynamic and Mikelić [7], Koch et al. [8] and Cao et al. [9] for the study of two-phase flow equations with a dynamic capillary pressure.

In [7], A. Mikelić investigates one pseudoparabolic equation. This equation describes unsaturated flows in porous media with dynamic capillary pressure–saturation relationship. In general, such models arise in a number of cases when non-equilibrium thermodynamics or extended non-equilibrium thermodynamics are used to compute the flux. The equation considered in [7] corresponds to the saturation equation in the case where the global pressure is considered to be a constant.

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The author proves firstly the existence of at least one weak solution to the regularized equation (permeability and capillary pressure are regularized) and next he lets the regularization parameter goes to zero to show the existence of weak solutions under degenerate case.

In [9], the authors investigate elliptic–parabolic system describing the flow of two incompressible, immiscible fluids including dynamic effects in the phase–pressure difference. The authors analyse the existence and uniqueness of a weak solution to the system. They consider power law for permeabilities, capillary pressure and damping coefficient. Under specific relationships between the power coefficient, they establish the existence of a weak solution. The present system is similar to the one studied in [9]; we treat the nondegenerate case thus a weak solution as defined in Definition 1 is more regular than proposed in their work.

In [8] Koch et al. investigate the motion of two immiscible fluids in the context of dynamic capillary hysteresis; they derive an existence result for the hysteresis two phase model for non-degenerate permeability and capillary pressure curves. The system is treated in the formulation of two pressures whereas in [9] the formulation of global pressure–saturation is considered. The authors use a Galerkin method to establish the existence result in which the pressures solve an elliptic system.

A large variety of methods have been proposed for the discretization of parabolic systems modeling the displacement of immiscible incompressible two-phase flows with static capillary pressure in porous media. We refer to [10] and [11] for the finite difference method. The finite volume methods have been proved to be well adapted to discretize conservative equations. The cell-centered finite volume scheme has been studied in the pioneer work in [12]. For equilibrium capillarity case and the two-phase incompressible immiscible flows, the convergence of a cell-centered finite volume scheme to a weak solution is studied in [13]. For this model, the finite elements scheme is studied in Chavent et al. [14]. Recently, the convergence analysis of a finite volume scheme for a degenerate compressible and immiscible flow in porous media has been studied by Bendahmane et al. [15] when the densities of each phase depend on the global pressure, by B. Saad and M. Saad [16] for the complete system when the density of the each phase depends on their own pressure and by Saad et al. [17] for compositional compressible degenerate two-phase flow in saturated–unsaturated heterogeneous porous media.

In order to provide accurate simulations in the case of dynamic capillary pressure, several numerical methods have been proposed in the literature, including a finite difference method with minmod slope limiter in Van Duijn et al. [18], Godunov-type staggered central schemes in Wang and Kao [19], two semi-implicit schemes based on equivalent reformulations in Van Duijn et al. [5], finite volume scheme in [20], finite element scheme in Koch et al. [8].

In this paper, we investigate a TPFA (two points flux approximation) finite volume scheme to approximate the solutions of the nondegenerate two-phase flow with dynamic capillary pressure. The proposed finite volume scheme is written in a general form as a discrete variational formulation. This scheme is mainly based on the particular choice of the mobilities on the interfaces or equivalently from the definition of mobilities on the dual mesh. In fact, we define mobilities on interfaces as the mean value of the inverse of mobilities. This choice permits to derive nonstandard energy estimates by considering nonlinear test function. The use of nonlinear test function is widely used to prove existence of solutions, we cite for instance [7] and [9] for two incompressible phase flow with dynamic capillary pressure and [16,21–23] for compressible two-phase flow in porous media.

The rest of the paper is organized as follows. Section 2 is devoted to the introduction of the model and to the statement of the main convergence result. In Section 3, we introduce some notations for the finite volume method and we present the construction of our numerical scheme. In Section 4, we derive the main a priori estimates on discrete global pressure and on the discrete saturation. Section 5 is devoted to the well posedness of the scheme. Section 6 is devoted to a space–time L^1 compactness of sequences of approximate solutions. Finally, the passage to the limit on the scheme and convergence is achieved in Section 7.

2. Mathematical formulation of the continuous problem

Consider a porous medium saturated with a fluid composed of two immiscible phases (liquid and gas). We refer to [23] for more details. Let $T > 0$ be the final time fixed, and let be Ω a bounded open subset of $\mathbb{R}^\ell$ ($\ell \geq 1$). We set $Q_T = (0, T) \times \Omega$, $\Sigma_T = (0, T) \times \partial\Omega$. In order to define the physical model, we write the *mass conservation* of each component in each phase

$$\phi\partial_t s_l + \mathrm{div}\mathbf{V}_l = 0 \tag{1}$$

$$\phi\partial_t s_g + \mathrm{div}\mathbf{V}_g = 0. \tag{2}$$

Here the subscript l and g represent respectively the liquid phase and the gas phase. Quantities ϕ, s_α, and $\mathbf{V}_\alpha$ represent respectively the porosity of the medium, the saturation of the α phase and the velocity of the α phase, for $\alpha = l, g$. The velocity of each fluid $\mathbf{V}_\alpha$ is given by the Darcy law:

$$\mathbf{V}_\alpha = -\mathbf{K}\frac{k_{r_\alpha}(s_\alpha)}{\mu_\alpha}\nabla p_\alpha, \qquad \alpha = l, g \tag{3}$$

where $\mathbf{K}$ is the permeability tensor of the porous medium, k_{r_α} the relative permeability of the α phase, μ_α the constant α-phase's viscosity and p_α the α-phase's pressure. For more clarity of the paper, the gravity term neglected. Assuming that the phases occupy the whole pore space, the phase saturations satisfy

$$s_l + s_g = 1. \tag{4}$$

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