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Effective low-Mach number improvement for upwind schemes

Shu-sheng Chen [a], Chao Yan [a,*], Xing-hao Xiang [b]

[a] School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China
[b] China Aerodynamics Research and Development Center, Mianyang 621000, China

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ABSTRACT

In this paper, we present an effective low-Mach number improvement for upwind schemes. The artificial viscosity of upwind schemes scales with $1/Ma$ incurring a loss of accuracy for the Mach number approaching zero. The remedy is based on three steps: (i) the jump of the left and right states is split into the density diffusion part and velocity diffusion part; (ii) the velocity diffusion part is rescaled by multiplying a scaling function; (iii) the scaling function is only related to the local Mach number without the cut-off reference Mach number and meantime restricted by a shock senor. The resulting modification is very easily implemented and applied within Roe, HLL and Rusanov, etc. Then, asymptotic analysis and numerical experiments for a wide Mach number demonstrate that this novel approach is equipped with these attractive properties: (1) free from the cut-off global problem and damping checkerboard modes; (2) satisfying the correct Ma^2 scaling of pressure fluctuations, the divergence constraint and a Poisson equation; (3) independent of Mach number in terms of accuracy; (4) better accurate and higher resolution in low Mach number regimes when compared with existing upwind schemes, and applicable for moderate or high Mach numbers. Thus, the proposed modification is expected to provide as an excellent and reliable candidate to simulate turbulent flows at all Mach numbers.

1. Introduction

Nowadays, upwind schemes have undergone extensive development and prevailed in the simulation of compressible flows. Among them, Roe [1], HLL [2] and Rusanov [3] schemes are the most widely-used approximate Riemann solver. Roe scheme is very applicable for its high discontinuity and boundary resolutions. Unfortunately, it suffers from the disastrous carbuncle phenomenon [4] and violates the entropy condition. HLL scheme is very efficient and robust against shock anomalies. It can exactly capture an isolated shock and preserve positivity, whereas it fails to resolve contact discontinuity exactly. Rusanov scheme is the simplest approximate Riemann solver. It has the similar properties as HLL, but generates more numerical dissipation than HLL and Roe schemes. To combine the merits and avoid the demerits of these schemes, the hybrid method has been extensively investigated. For instance, Nishikawa et al. [5] applied Rusanov/HLL solver in the direction normal to shock and Roe solver across shear layers, resulting in the rotated-hybrid Riemann solvers with carbuncle-free and boundary-layer-resolving properties.

However, using these upwind schemes in low Mach number limit faces two major deficiencies [6,7]: difficult convergence and deteriorated accuracy. Moreover, many applications involve both compressible and nearly incompressible flows in a single domain, such as high-speed flows with large embedded low-speed regimes in the separation and near-wall regions.

* Corresponding author.
E-mail addresses: sshengchen@outlook.com (S.-s. Chen), yanchao@buaa.edu.cn (C. Yan).

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It is observed that low-speed resolution has a significant effect in the accurate prediction of turbulent flows, but upwind schemes heavily dampen the low Mach number features [8]. This explains that upwind schemes are not reliable enough for a wide range of applications in the existing code allows. Therefore, many investigators make great efforts to extend the compressible schemes to efficiently calculate very low Mach number flows as well as maintain shock-capturing capability.

To address the accuracy and stiffness problems, preconditioning techniques have been developed by Turkel [9,10], Weiss and Smith [11] and Choi and Merkle [12], etc. over the past years. Preconditioned Roe (P-Roe) [11,13,14] and preconditioned HLL (P-HLL) [15] schemes were proposed. Preconditioned methods alter the acoustic speed of governing equations and balance the propagation of wave speeds to alleviate the stiffness. The viscosity matrix is correspondingly scaled to improve the accuracy. The main drawback is that preconditioning techniques need at least one artificial parameter, such as the reference Mach number Ma, and encounter the global cut-off problems [13]. In addition, preconditioned methods do not satisfy the divergence constraint of the 0-order velocity field and the Poisson equation of the 2-order pressure field [16].

On the other hand, the accuracy problem is inherently relevant to the numerical flux scheme as reported by [7]. Guillard and Viozat [14] performed asymptotic analysis to reveal that the numerical viscosity of upwind schemes scales with $1/Ma$ on a mixed Cartesian grid for $Ma \to 0$, giving rise to a loss of accuracy and the incorrect scaling of pressure and density fluctuations. This shows that upwind schemes support pressure fluctuations of order Ma in low Mach number, which is significantly different from the Euler system that supports pressure fluctuation of order Ma^2 [14]. Afterwards, a variety of improvements for upwind schemes has been proposed for all Mach numbers. Based on the reconstruction process, Thornber et al. [8] proposed a simple local modification of the reconstructed velocity variables to reduce the numerical dissipation and improve the low-Mach number resolution. In terms of Riemann solver, the improved Roe-type schemes (e.g. Thornber–Drikakis's Roe [17] and Fillion's Roe [18], All-Speed Roe [16], Rieper's Roe [19], etc.) have been devised without the accuracy problem and the global cut-off problem for the past few years. In low Mach number regime, there is the large disparity between the acoustic speed and the convective speed. Thornber and Drikakis [17] modified the acoustic speed term of the right eigenvector matrix to eliminate this large disparity. Li and Gu [16] proposed an all-speed Roe-type scheme (A-Roe) by fixing the acoustic wave strength of the non-linear eigenvalues. A-Roe scheme is very simple and has the superior asymptotic properties than P-Roe scheme, but suffers from the serious checkerboard problem. The interpolation techniques [20] are applied in A-Roe scheme to damp checkerboard modes, but also bring the artificial parameters. It is also unknown whether the interpolation techniques have an impact for moderate or high Mach number flows. In contrast, Rieper [19] modified the artificially large normal velocity component to accomplish LMRoe scheme that is very similar to Thornber's reconstruction method. Rieper's low-Mach number fix automatically suppresses checkerboard modes and is very general. Nevertheless, it fails to extend some upwind schemes, like HLL, Rusanov, Rotated-Roe–HLL [5], Rotated-Roe–Rusanov [5], etc., due to the excessive artificial dissipation on the entropy and shear waves [19].

In the current study, we suggest another simple and efficient improvement to extend upwind schemes towards a unified formulation that reliably handle low Mach number flows, and meantime keep applicable for moderate or high Mach number flows. We split the jump of the left and right states into the density diffusion part and velocity diffusion part. The proposed modification is built on rescaling the velocity diffusion part by multiplying the scaling function, which is different from the previous work [8,16–19]. Since the introduced scaling function affects stability and accuracy, we pay specific attention to its definition. Then, asymptotic analysis is performed to uncover the superior low Mach number behaviors of this modification like the continuous Euler system. We want to emphasize the major advantages of our approach here: (1) Like preconditioning techniques, A-Roe scheme, LMRoe scheme, etc., this approach recovers the correct Ma^2 scaling of pressure fluctuations and the solution converges to a reasonable approximation of incompressible flow for Mach number vanishing; (2) Unlike preconditioning techniques but as LMRoe and A-Roe schemes, this approach is free from the global cut-off problems, and satisfies the divergence constraint of the 0-order velocity field and the Poisson equation of the 2-order pressure field under the isentropic condition; (3) Unlike A-Roe scheme but as LMRoe scheme and preconditioning techniques, this approach can damp checkerboard mode; (4) Compared to LMRoe scheme, this approach is more general and successfully applied in more dissipative schemes, such as HLL, Rusanov, Rotated-Roe–HLL, etc. In addition, the proposed modification is with a truncation error $O(\delta)$ independent of Mach number in terms of accuracy. Through testing massive numerical cases, one objective of the effort is to further study the performance of numerical flux scheme at low speeds, especially in the simulation of turbulent flows.

This paper is organized as follows. In Section 2, the governing equations and Riemann solvers are briefly overviewed. In Section 3, the low-Mach number improvement is accomplished. In Section 4, we perform asymptotic analysis to reveal low Mach number behavior of the proposed modification. In Section 5, numerical cases covering a wide range of applications are carried out. Finally, concluding remarks are given in Section 6.

2. Governing equations and Riemann solvers

2.1. Governing equations

Consider the ideal two-dimensional Euler system of conservation laws:

$$\begin{aligned} &\frac{\partial \boldsymbol{Q}}{\partial t} + \nabla \cdot \boldsymbol{E}(\boldsymbol{Q}) = 0 \\ &\boldsymbol{E}(\boldsymbol{Q}) = [\boldsymbol{F}(\boldsymbol{Q}), \boldsymbol{G}(\boldsymbol{Q})]^T \end{aligned} \tag{1}$$