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Finite Pointset Method for biharmonic equations

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ABSTRACT

Finite pointset method is one of the grid free methods that is used to solve differential equations arising from physical problems. It is a local iterative procedure based on weighted least square approximation technique. In this paper, biharmonic equation with simply supported, clamped and Cahn–Hilliard type boundary conditions, is solved using the finite pointset method. Numerical examples illustrate the efficiency of the method.

1. Introduction

In many engineering applications such as bending of a thin plate and flow of a viscous fluid, the biharmonic equation plays a crucial role. Numerical solution for these kinds of practical problems has been addressed, in the literature, with help of classical numerical methods such as the finite difference methods, finite element methods, and boundary element methods. The main success of these classical methods depends on their ability to construct and to maintain grids throughout the numerical simulations. If the geometry of the problem is complex, one may experience distortion in the numerical grid. More computational effort is required for a better accuracy in such situations. Many grid free approaches have been proposed and developed to overcome the disadvantages that may arise in the grid based methods [1,2].

The Finite Pointset Method (FPM) had first been developed by Tiwari et al. [3]. The basic idea of this method is to approximate spatial derivatives of a function at an arbitrary point from the function values at the surrounding cloud of points. These points need not be regularly distributed and can even be quite arbitrary. In [1], numerical solution for conservation laws is presented using two meshfree methods by finite volume particle method and finite pointset method. The computational speed-up and efficiency of a GPU versus CPU using the finite pointset method is presented in [4]. The authors in [5] present a finite pointset method based on the projection technique for the incompressible Navier–Stokes equations. In [6], the numerical simulations for liquid–liquid two phase flow fields in an extraction column are presented using finite pointset method.

In this paper, the biharmonic equation is solved numerically using a finite pointset grid free method. The present method is a local iterative process based on the least square approximation. Moreover at every iteration step, the biharmonic equation with boundary condition is approximated at each particle using the information of solution function at neighboring cloud of points. The boundary conditions can easily be handled because they can be replaced by a discrete set of boundary values at boundary particles.

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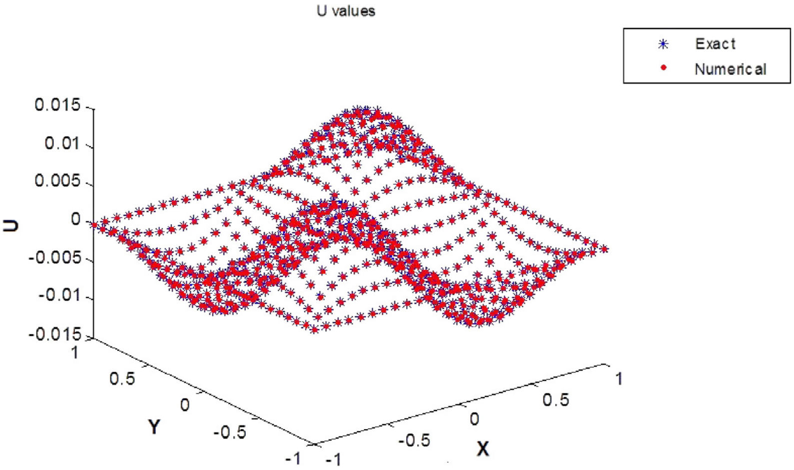

Fig. 4.1. *U* values for Uniform Distribution of points for Simply supported boundary condition.

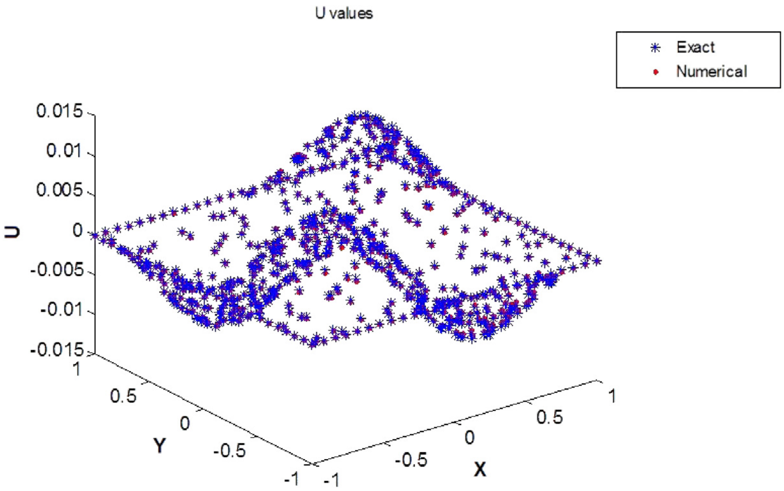

Fig. 4.2. *U* values for Non-uniform Distribution of points for Simply supported boundary condition.

The organization of the paper is as follows: In Section 2, the numerical method is described for the biharmonic equation. In Section 3, an algorithm for the finite pointset method for solving the biharmonic equation is provided. In Section 4, some numerical experiments and convergence analysis are presented.

2. Grid free method for solving general biharmonic equations

Let us consider the following biharmonic equation which is a linear partial differential equation of the fourth order:

$$\Delta^2 u - a\Delta u + bu = f \qquad in \qquad \Omega, \tag{2.1}$$

with simply supported boundary conditions

$$u = g_0 \quad , \quad \frac{\partial^2 u}{\partial \vec{n}^2} = g_1 \qquad on \qquad \Gamma, \tag{2.2}$$

(or) with clamped boundary conditions

$$u = g_0 \quad , \quad \frac{\partial u}{\partial \vec{n}} = g_1 \qquad on \qquad \Gamma, \tag{2.3}$$