ARTICLE IN PRESS
Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

General decay of a von Karman plate equation with memory on the boundary

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ARTICLE INFO

Article history:
Received 22 December 2016
Accepted 21 January 2018
Available online xxxx

Keywords:
Karman equation
General decay
Convexity
Memory term

ABSTRACT

We investigate the influence of boundary dissipation on the decay property of the solutions for a von Karman plate equation with a memory condition on one part of the boundary. Dropping the condition $u_0 = 0$ on one part of the boundary, we show a general stability result for the equation via setting modified energy functionals which are equivalent to the energy of the equation and using some properties of convex functions. This result allows a wider class of relaxation functions and improve earlier results of Mustafa and Abusharkh (2015) and Park and Park (2005).

1. Introduction

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\Omega := \Gamma$, Γ_0 and Γ_1 have positive measures and satisfy $\Gamma_0 \cup \Gamma_1 = \Gamma$, $\Gamma_0 \cap \Gamma_1 \neq \emptyset$, and $x = (x_1, x_2)$. Let us denote the external unit normal vector on Γ by $\nu = (\nu_1, \nu_2)$ and the corresponding unit tangent vector by $\tau = (-\nu_2, \nu_1)$.

We investigate the following von Karman plate equation with a memory condition on a part of the boundary:

$$u_{tt}(x,t) + \Delta^2 u(x,t) = \alpha[u(x,t), F(u(x,t))] \text{ in } \Omega \times (0,\infty), \tag{1.1}$$

$$\Delta^2 F(u(x,t)) = -[u(x,t), u(x,t)] \text{ in } \Omega \times (0,\infty), \tag{1.2}$$

$$F(u(x,t)) = \frac{\partial F(u(x,t))}{\partial \nu} = 0 \text{ on } \Gamma \times (0,\infty), \tag{1.3}$$

$$u(x,t) = \frac{\partial u(x,t)}{\partial \nu} = 0 \text{ on } \Gamma_0 \times (0,\infty), \tag{1.4}$$

$$\frac{\partial u(x,t)}{\partial \nu} + \int_0^t g_1(t-s)\mathcal{B}_1 u(x,s)ds = 0 \text{ on } \Gamma_1 \times (0,\infty), \tag{1.5}$$

$$u(x,t) - \int_0^t g_2(t-s)\mathcal{B}_2 u(x,s)ds = 0 \text{ on } \Gamma_1 \times (0,\infty), \tag{1.6}$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x) \text{ in } \Omega, \tag{1.7}$$

where $\alpha > 0$, the von Karman bracket $[w, \phi]$ is given by

$$[w,\phi] \equiv w_{x_1x_1}\phi_{x_2x_2} + w_{x_2x_2}\phi_{x_1x_1} - 2w_{x_1x_2}\phi_{x_1x_2},$$

$$\mathcal{B}_1 u = \Delta u + (1-\mu)B_1 u,\ \mathcal{B}_2 u = \frac{\partial}{\partial\nu}\Delta u + (1-\mu)B_2 u,$$

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https://doi.org/10.1016/j.camwa.2018.01.032

here

$$B_1u = 2\nu_1\nu_2 u_{x_1x_2} - \nu_1^2 u_{x_2x_2} - \nu_2^2 u_{x_1x_1},$$
$$B_2u = \frac{\partial}{\partial\tau}[(\nu_1^2 - \nu_2^2)u_{x_1x_2} + \nu_1\nu_2(u_{x_2x_2} - w_{x_1x_1})],$$

and the constant μ, $0 < \mu < 1/2$, represents Poisson's ratio.

When $\alpha = 0$ in problem (1.1)–(1.7), Santos and Junior [1] showed existence of the solutions and an exponential(polynomial) energy decay rate under the conditions

$$k_i \in C^2(\mathbb{R}_+), k_i(0) > 0, k_i'(t) \le -c_1k_i(t), k_i''(t) \ge -c_2k_i'(t), i = 1, 2, \tag{1.8}$$

where k_i denotes the resolvent kernel of $-\frac{g_i'}{g_i(0)}$ and c_i is positive constant. Mustafa and Abusharkh [2] extended the results in [1] by considering general decay rates of the energy for the same problem under the generalized conditions

$$k_i(0) > 0, \lim_{t\to\infty} k_i(t) = 0, k_i'(t) \le 0, k_i''(t) \ge H(-k_i'(t)), \tag{1.9}$$

where H is a positive function, which is linear or strictly increasing and strictly convex of class C^2 on $(0, r]$, $r < 1$, with $H(0) = 0$. Although this allows a wider class of relaxation functions and generalizes existing results in the literature, they imposed some conditions on the initial data such as $u_0 \equiv 0$ on a part of the boundary Γ in order to get the desired result.

When $\alpha \neq 0$, Park and Park [3] proved existence of the solution and an exponential decay rate of the energy under the assumption (1.8). They gave some restriction on α to overcome the difficulty generated by geometry conditions and the von Karman bracket $\alpha[u, F(u)]$. In the case $\alpha \neq 0$, problem (1.1)–(1.3) becomes von Karman plate equations. Existence, regularity, energy decay, and attractor for von Karman plate equations were considered by many author (see e.g. [4–8]). For von Karman plate equations of memory type, we refer readers to [3,9–15] and references therein.

On the other hand, Chen et al. [16] considered a transmission problem for degenerate equation with boundary memory conditions under the assumption (1.9). They established a general decay result by dropping some conditions imposed on initial data u_0. Inspired these works, we consider a general decay of the energy for problem (1.1)–(1.7) when the resolvent kernel k_i satisfies (1.9) and the initial data u_0 is not imposed the hypothesis $u_0 = 0$ on a part of the boundary. To get our desired result, we use the idea given in [2,16,17] with some necessary modification due to the nature of the problem treated here. It is worth to mention that many authors have been interested in investigating general decay results by weakening conditions of the kernel functions. For the related problems, we also refer [3,18–22] and references therein.

The plan of this paper is as follows. In Section 2, we give some notations and material needed for our work. In Section 3, we derive general decay estimates of the energy.

2. Preliminaries

In this section, we present some material needed in the proof of our results. Throughout this paper, we denote

$$(w, \phi) = \int_\Omega w(x)\phi(x)dx, (w, \phi)_{\Gamma_1} = \int_{\Gamma_1} w(x)\phi(x)d\Gamma,$$

and define

$$W := \{w \in H^2(\Omega) : w = \frac{\partial w}{\partial\nu} = 0 \text{ on } \Gamma_0\},$$
$$\tilde{W} := \{w \in H^2(\Omega) : w = \frac{\partial w}{\partial\nu} = 0 \text{ on } \Gamma_1\}.$$

Let us denote $\|\cdot\|_X$ the norm of a Banach space X. For brevity, we denote $\|\cdot\|_{L^2(\Omega)}$ and $\|\cdot\|_{L^2(\Gamma_1)}$ by $\|\cdot\|$ and $\|\cdot\|_{\Gamma_1}$, respectively.

The integration by parts gives [23]

$$(\Delta^2 w, \phi) = \int_\Omega a(w, \phi)dx - (\mathcal{B}_1 w, \frac{\partial\phi}{\partial\nu})_\Gamma + (\mathcal{B}_2 w, \phi)_\Gamma, \tag{2.1}$$

where the bilinear symmetric form $a(w, \phi)$ is given by

$$a(w, \phi) = w_{x_1x_1}\phi_{x_1x_1} + w_{x_2x_2}\phi_{x_2x_2} + \mu(w_{x_1x_1}\phi_{x_2x_2} + w_{x_2x_2}\phi_{x_1x_1}) + 2(1-\mu)w_{x_1x_2}\phi_{x_1x_2}.$$

Because $\Gamma_0 \neq \emptyset$, we know (see [5]) that $\int_\Omega a(w, w)dx$ is equivalent to the $H^2(\Omega)$ norm on W, that is, there exists $c_i > 0$, $i = 1, 2$, satisfying

$$c_1\|w\|^2_{H^2(\Omega)} \le \int_\Omega a(w, w)dx \le c_2\|w\|^2_{H^2(\Omega)}. \tag{2.2}$$

Please cite this article in press as: S.-H. Park, General decay of a von Karman plate equation with memory on the boundary, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.01.032.