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A novel active contour model for medical images via the Hessian matrix and eigenvalues

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ABSTRACT

This paper presents a new level set formulation for active contour models (ACM). We propose the idea of integrating the eigenvalue information of Hessian matrix into the level set function. By this new level set function, the principal curvature information of images is used to enhance the ability of segmenting boundary regions. The advantages of our model are as follows: firstly, the interior and exterior object boundaries can be segmented with the initial contour being anywhere in the input image. Secondly, this method can work with heterogeneous images. Thirdly, the proposed model can produce smooth and right boundaries of objects having vital importance in medical operations. Extensive experiments demonstrate that the proposed model can obtain better segmentation results.

1. Introduction

The segmentation is one of the necessary image processing techniques receiving significant attention in various computer vision applications including object recognition and classification. Active Contour Model (ACM) is one of the most popular segmentation techniques. The major consideration of ACM is to evolve a closed curve in order to obtain the correct object boundaries. Depending on how object boundary is detected, the ACMs can be categorized into parametric [1–5] and geometric models [6–17]. Parametric active contours (or snake) are based on an energy minimization, while the geometric active contours based on the theory of curve evolution. In general, geometric active contour models are given in a level set formulation.

The original active contour model was firstly proposed by Kass et al. [1], in which a parametric representation of the contour was used. This model has been known as energy-minimizing curves or snakes. However, it has significant drawbacks such as: (i) it needs prior information of the object boundary, (ii) it is easily affected by noise in the input image, (iii) it has difficulties caused by intensity inhomogeneities. With the improvement of active contour methodology, a large number of active contour models have been proposed to overcome such drawbacks. To address some of the key problems in the original model, Cohen [2] has modified the external forces that push the curve to the boundaries. In this model, the curve behaves like a balloon which is inflated by an additional force. Moreover, the contour curve passes over discontinuous boundary and is stopped only if the boundary is strong. One of the most efficient parametric active contours models is the Gradient Vector Flow (GVF) approach [3,4]. Xu and Prince [4] have used an external field to enhance the convergence to weak and long boundary structures. Ren et al. [5] proposed a new parametric GVF model based on the augmented Lagrangian method. They reformulated GVF model as a convex optimization problem, and used an effective optimization scheme. Moreover,

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they adopted the multiresolution approach to reduce the computational cost. Li et al. [6] proposed a region-based ACM in a variational level set formulation to deal with intensity inhomogeneity. They defined a region-scalable fitting (RSF) energy functional. RSF energy functional contains a contour and two fitting functions. Fitting functions were used to approximate the image intensities on the two sides of the active contour curve. RSF model provides the use of intensity information in image regions at a specific scale, but it is known sensitive to initialization. Recently, Zhang et al. [7] developed a level set active contour model (LSACM) for medical image segmentation in the presence of intensity inhomogeneity. In this model, the inhomogeneous objects are modeled as Gaussian distributions of different variances and means. Their model defines a mapping from the input image domain to another domain. To map the input image into another domain, a sliding window method is utilized. Since the intensity distribution of inhomogeneous object is Gaussian, these objects are more easily segmented. Finally, maximum likelihood energy functional is defined on the image regions. This functional combines the level set function, the bias field and the piecewise constant function approximating the original image.

Different from parametric active contour models, geometric active contour models have been developed in the literature. In general, image gradients have been used in these models to detect the edges of objects. Caselles et al. [13] developed a geometric active contour method based on curve evolution. This model includes a comprehensive mathematical analysis. To evolve the contour curve in the direction of normal force, energy based function is used. The contour curve is stopped at the object boundary. In [14], geometric active contour was proposed by using level set formulation. In this work, they adopt level set methods to the problem of shape recovery. With this approach, complex shapes can be detected from given images. However, this approach cannot converge to the object boundary accurately for images having low contrast between the background and the objects. To overcome such drawbacks, the geodesic active contour (GAC) [15] was developed. The GAC model uses gradient computation to construct an Edge Stopping Function (ESF). Moreover, the GAC model allows to connect geometric active contours based on the theory of curve evolution and traditional snakes based on energy minimization. The GAC is usually sensitive to weak edges and noise. Holtzman-Gazit et al. [16] developed a variational model, which uses a function including a geodesic active contour (GAC) term and edge-based term. The developed level set includes a minimal variance term that measures the homogeneity inside and outside the object. A robust numerical algorithm for the proposed model was developed in order to accelerate its convergence. Li et al. [18] proposed a new variational level set formulation called Distance Regularized Level Set Evolution (DRLSE). The proposed level set formulation provides stable level set evolution and efficient numerical scheme. Zhang et al. [19] developed an ACM which is carried out with a novel level set formulation. This model is implemented with a different processing named Selective Binary and Gaussian Filtering Regularized Level Set (SBGFRLS) model. The ACM with SBGFRLS utilizes the statistical pixel information of object regions to constitute a region-based Signed Pressure Force (SPF) function. As an alternative to ACM with SBGFRLS model, Talu [20] developed an online region-based active contour method named ORACM. The ORACM utilizes a user-defined active contour and then continuously updates it. Although the ACM with SBGFRLS and ORACM methods have some significant benefits, they cannot produce smooth object boundaries, and this tends to undesired active contours.

All the above reviewed geometric active contour models generally use image gradient calculation to attract the contour curve toward object boundaries. Since direct-gradient-based models depend on edge information, they have poor convergence to discontinuous boundary and noise. These models can benefit from a balloon force function to shrink or expand the active contour. In order to achieve better segmentation performance, the initial active contour curve must be located near the object edges, especially when complex boundary and strong noise are presented in the image [21,22].

On the other hand, some of geometric models cannot detect the discrete and weak object boundaries. To avoid this drawback, the Chan–Vese (CV) model [8] finds the correct object boundary in a variational level set formulation with intensity information, and it does not use the gradient information in level set formulation. The level set formulation of the CV model uses a region based energy function, which is derived from the Mumford–Shah model [23]. However, the CV model has much convergence range, and it does not work well for images with intensity inhomogeneity. Inspired by the C–V model, different ACMs have been proposed and used in several image processing problems [24–26]. In [26], Zhang et al. presented robust region based ACM using local image statistics. These statistics are calculated using local image fitting energy (LIF). Moreover, Gaussian filtering for variational level set has been used to regularize the level set function. Paragios and Deriche [27] proposed a boundary and region-based model under a curve-based optimization function. The proposed model has been obtained by unifying boundary-based and region information as an improved GAC model. The objective function is optimized using a gradient-descent algorithm. To obtain accurate cell boundaries, Ersoy et al. [28] proposed a modified geodesic active contour model. In this model, the level set evolution is performed by using a new adaptive stopping function. In [29], an efficient level set based active contour model has been developed to integrate both region and boundary information. This model uses hybrid level set formulation based on region term and curvature term. Especially, the region term developed in the level set formulation has the important advantages over the CV model in dealing with medical images with low contrast [29].

Some of the ACM models use gradient information of the current level set as a speed-controlling function [30]. When the active contour moves through the object boundaries, its speed can slow down, and produces rough object boundaries. This process causes very high computational cost. Moreover, the gradient computation along the weak object boundaries may cause a problem since the value of gradient vector here is nearly zero.

In this paper, we propose a novel geometric active contour model based on the eigenvalue analysis of Hessian matrix. Hessian matrix and its eigenvalues have been used in segmentation and enhancement of medical images, especially in the case of vessel images, which have thin and long structures that are difficult to distinguish [31]. In particular, the shape Hessian